Universality in Fixed Trace Ginibre Ensembles

Milan Cikovic, Gernot Akemann

Faculty of Physics, Bielefeld University, Germany

Universität Bielefeld

Introduction

- The **Ginibre ensemble** defined by

$$\mathbf{Z}_G := \int_{\mathbb{C}} \mathrm{e}^{-\mathrm{Tr}\left[\mathbf{G}\mathbf{G}^{\dagger}\right]} \, \mu\left(\mathrm{d}\mathbf{G}\right)$$

deals with quadratic matrices of size $\mathbf{N}$ $\quad \mathbb{N}$ with no hermitian or unitary conditions imposed and was introduced by Ginibre [1] in 1965. Here we are interested in the case of complex matrices $\mathbf{G}$ $\quad \mathbb{C}^{\mathbf{N}\times\mathbf{N}}$ where $\mathbf{G}_{\mathbf{ij}}$ $\quad N\ (\mathbf{0},\mathbf{1})$ for the real and imaginary part, respectively.
Using the Schur decomposition, computing the Jacobian, following the method of orthogonal polynomials (monoms) and applying the famous Dyson theorem leads to the **k**-point function in terms of the determinant of the kernel. In particular the angle-independent normalized eigenvalue distribution in the complex plane is given by the one point function

$$\mathbf{R}_{\mathbf{1},G}\left(\mathbf{r}\right) = \frac{\mathbf{1}}{\mathbf{2}\pi\mathbf{N}} \mathrm{e}^{-\mathbf{r}^2} \sum_{\mathbf{k}=\mathbf{0}}^{\mathbf{N}-\mathbf{1}} \frac{\mathbf{r}^{\mathbf{2k}}}{\mathbf{k}!}.$$

The initial interests in non-hermitian matrices goes back to the theory of scattering quantum chaotic systems [2] whose particles can escape at a given energy to infinity or come from infinity. The random matrix description comes with the so called Heidelberg approach where the description of the theory is based on an effective Hamiltonian which is non-hermitian.
- The **fixed trace ensemble** is defined by

$$\mathbf{Z}_{\delta} := \int_{\mathbb{C}} \delta\left(\mathbf{t} - \mathrm{Tr}\left[\mathbf{G}\mathbf{G}^{\dagger}\right]\right) \, \mu\left(\mathrm{d}\mathbf{G}\right),$$

where $\mathbf{G}$ preserve the above properties.
The fixed trace ensemble, in particular for covariance matrices $\mathbf{W} = \mathbf{X}\mathbf{X}^{\dagger}$, $\mathbf{X}$ $\quad \mathbb{C}^{\mathbf{N}\times\mathbf{M}}$ can be used to describe the entanglement of a bipartite quantum system [3]. Let us consider a system A in which we are interested in and B as the environment. The bipartite Hilbert space is give by