IDENTITIES AND EXPONENTIAL BOUNDS FOR TRANSFER MATRICES

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Abstract: Analytic statements can be made on eigenvalues z_i and singular values ${}_i$ of the transfer matrix T_n of a single general block tridiagonal matrix H:

1) duality identity and Thouless-like identities for $\frac{1}{n}\log|z_i|$ (exponents);

2) There are constants K, H such that

$$ {}_i > e^{Hn+K}, \qquad {}_{m+i} < e^{-Hn-K} \qquad i = 1...m$$

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Block tridiagonal matrix & its transfer matrix

$$H = \begin{bmatrix} A_1 & B_1 & & C_1 \\ C_2 & \ddots & \ddots & \\ & \ddots & \ddots & B_{n-1} \\ B_n & & C_n & A_n \end{bmatrix}_{nm\times nm}$$

$$H\Psi = E\Psi \qquad \Rightarrow \qquad T_n(E)\begin{bmatrix} {}_1 \\ {}_0 \end{bmatrix} = \begin{bmatrix} {}_{n+1} \\ {}_n \end{bmatrix}.$$

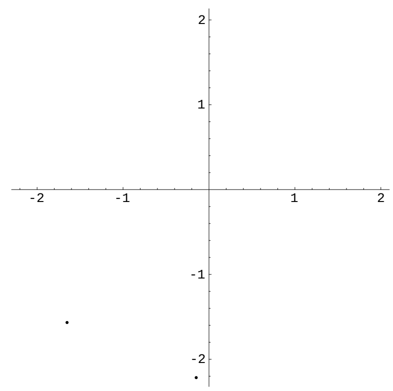
$${}_{n+1} = {}_1, \qquad {}_n = {}_0$$

$$T_n(E) = {}^{\;n}_{k=1}\begin{bmatrix} B_k^{-1}(E-A_k) & -B_k^{-1}C_k^\dagger \\ I_m & 0 \end{bmatrix}_{2m\times 2m}$$

The spectral duality

$$T_{\mathbf{n}}(E) \begin{bmatrix} 1 \\ 0 \end{bmatrix} = z \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \Rightarrow \quad {}_{\mathbf{n}+1} = z \;{}_{1}, \qquad {}_{\mathbf{n}} = z \;{}_{0}$$

Introduce the auxiliary matrix $H(z$



2
1
-2
-1
1
2
-1
-2

Demko-Moss-Smith

Lemma [Chebyshev]

Theorem DMSII is used to give estimates on the singular values of the transfer matrix, whose blocks may be represented as blocks of the resolvent of H with corners removed:

<u>transfer matrix & resolvent</u>

$$g(E) = \begin{bmatrix} E - A_1 & -B_1 & & 0 \\ -C_2 & \ddots & \ddots & \\ & \ddots & \ddots & -B_{\mathbf{n}-1} \\ 0 & & -C_{\mathbf{n}} & E - A_{\mathbf{n}} \end{bmatrix}^{-1}$$

$$\begin{aligned} T_{\mathbf{n}}(E) &= \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \\ &= \begin{bmatrix} -B_{\mathbf{n}}^{-1}(g_{1,\mathbf{n}})^{-1} & -B_{\mathbf{n}}^{-1}(g_{1,\mathbf{n}})^{-1} g_{1,1} C_1 \\ g_{\mathbf{n},\mathbf{n}}(g_{1,\mathbf{n}})^{-1} & g_{\mathbf{n},\mathbf{n}}(g_{1,\mathbf{n}})^{-1} g_{1,1} C_1 - g_{\mathbf{n},1} C_1 \end{bmatrix} \end{aligned}$$

<u>Exponential bounds for singular values $_{\mathbf{k}}$ of T</u>

Lemma Let $t_{\mathbf{k}}$ $k = 1...m$ be the singular values of the block T_{11} of $T_{\mathbf{n}}(E)$, then:

$$t_{\mathbf{k}} > \frac{1}{K} q^{-\mathbf{n}/2}$$

Use the properties, 1) interlacing property $_{\mathbf{k}} \geq t_{\mathbf{k}} \geq \ _{\mathbf{m}+\mathbf{k}}$
2) T^{-1} is a transes of the blo