A Riemann-Hilbert approach to the Muttalib-Borodin ensemble

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The model

The Muttalib-Borodin ensemble [2] with parameter $\theta > 0$ and weight function w is the following probability density function:

$$\frac{1}{Z_n}\prod_{j<k}(x_k - x_j)(x_k^\theta - x_j^\theta)\prod_{j=1}^{n} w(x_j), \quad x_j \geq 0.$$

We consider an n-dependent weight function

$$w(x) = x^\alpha e^{-nV(x)}$$

with $\alpha > -1$ and an external field V. The ensemble is a **determinantal point process**, it can be written as

$$\det\left(K_{V,n}^{\alpha,\theta}(x_i, x_j)\right)_{1\leq i,j\leq n}$$

where $K_{V,n}^{\alpha,\theta}(x,y)$ is the so-called correlation kernel.

Known result and main interest

Our main interest is to study the large n behavior of $K_{V,n}^{\alpha,\theta}(x,y)$. Borodin [2] computed the **hard edge scaling limit** for the Laguerre case, namely if $V(x) = x$, then

$$\lim_{n\to\infty}\frac{1}{n^{1+1/\theta}}K_{V,n}^{\alpha,\theta}\left(\frac{x}{n^{1+1/\theta}}, \frac{y}{n^{1+1/\theta}}\right) = \mathbb{K}^{(\alpha,\theta)}(x,y)$$

with limiting correlation kernel

$$\mathbb{K}^{(\alpha,\theta)}(x,y) = \theta y^\alpha \int_0^1 J_{\frac{\alpha+1}{\theta},\frac{1}{\theta}}(ux) J_{\alpha+1,\theta}((uy)^\theta)\, u^\alpha\, du$$

where

$$J_{a,b}(x) = \sum_{j=0}^{\infty}\frac{(-x)^j}{j!\,\Gamma(a+bj)}.$$

The same limit turns up in products of random matrices [1,4,5]. From these models and others [7] we know that the limit can be expressed in terms of **Meijer G-functions**.

By **universality** the limit is expected to hold for a much larger class of external fields V and our goal is to prove this.

[1]