Non-intersecting random paths

- Given a 1-D diffusion process with transition probability density y .
  Suppose independent copies $_1$ conditioned to
  - start at time at given points $_1$ $_2$ $\cdots$,
  - end at time at given points $_1$ $_2$ $\cdots$,
  - not intersect in the full time interval .
- Then the positions of the paths at given time are a determinantal point process. consequence of Karlin-McGregor (1959) theorem
- The correlation kernel takes the form

$$y \quad \sum_{=1} \quad y$$

where biorthogonal functions

Correlation kernel

- The correlation kernel for non-intersecting Brownian paths with two endpoints can be expressed in terms of the solution of the RH problem

$$y \qquad \frac{}{-\, y} \left(\quad {}_1\, y \qquad {}_2\, y \right) + y^{-1} + \begin{pmatrix} \\ \\ \end{pmatrix}$$

with weights $_1$ $e^{-\,(\;-\;)}$, $_2$ $e^{-\,(\;+\;)}$ where

depends on

for two weights: Bleher-Kuijlaars (2004)

extension to more than two weights: Daems-Kuijlaars (2004)

- The RH problem is then analyzed with the steepest descent method for RH problems of Deift and Zhou.

Non-intersecting squared Bessel paths

- **PROPOSITION:**

  In the confluent limit → , → , the positions of

Global regime 1

THEOREM on global regime

- **As** $\to \infty$

THEOREM on local regime

- The correlation kernel y has scaling limits as $\to \infty$
  - Bulk scaling near a point :

$$\frac{\mathrm{s\ n} \quad - y}{- y}$$

  - Soft edge scaling near a point or if :

$$\frac{\mathrm{A} \quad \mathrm{A} \quad y - \mathrm{A} \quad \mathrm{A} \quad y}{- y}$$

  - Hard edge scaling near if

Second transformation $X \mapsto U$

- Define the -functions $\int \log$ – and for certain constant matrices and ,

$$\mathrm{d} \quad \mathrm{g}\left(e^{-} \quad (\;)\; e \;(\; - \;)(\;)\; e \quad (\;)\right)$$

- Then we have jump matrices of the form

$$\begin{pmatrix} e^{-\, n\phi \;\; x} & x^{\alpha} & 0 \\ 0 & e^{-\, n\phi \; - \; x} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad x \in \Delta_1$$

$$\Bigg($$