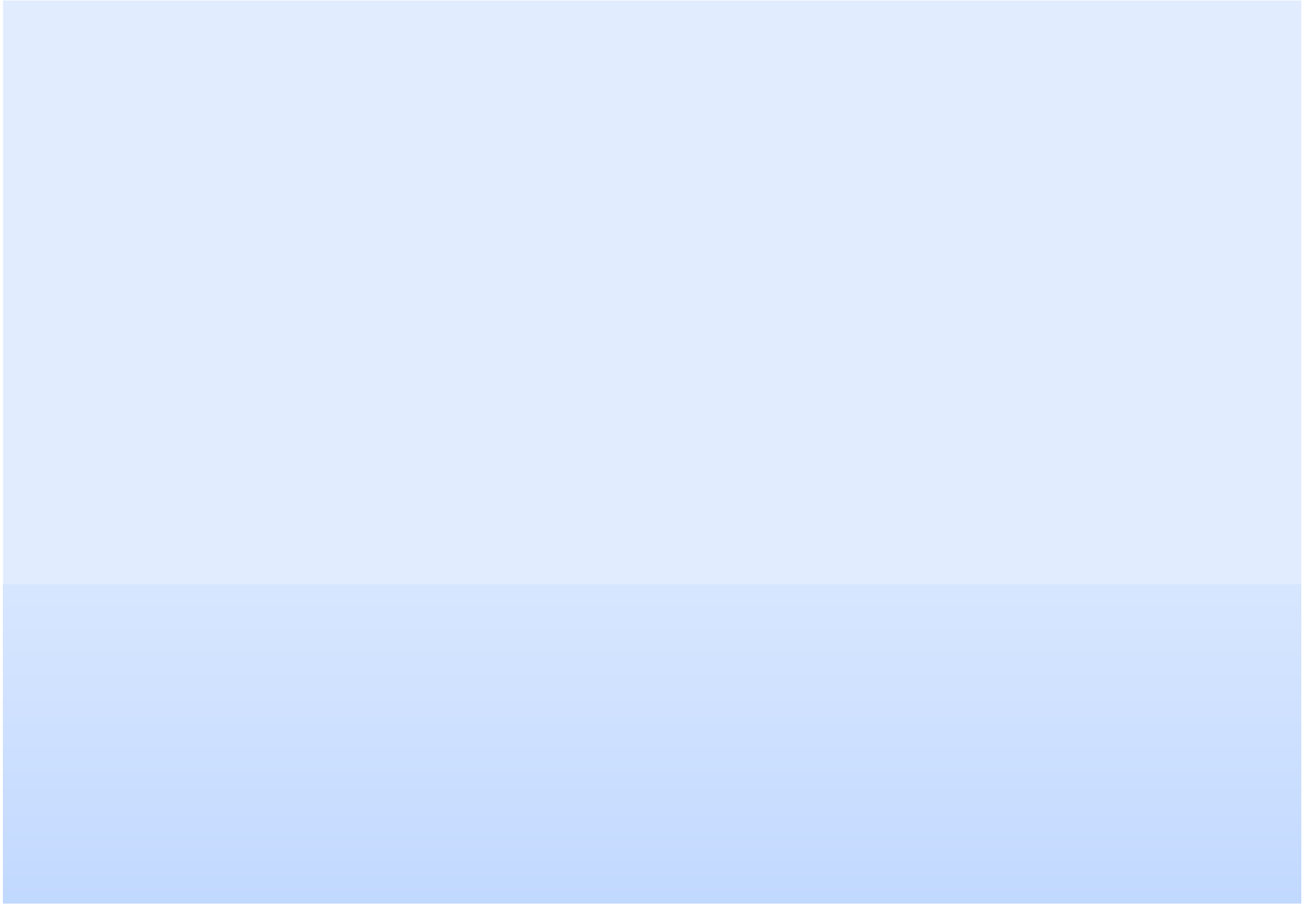


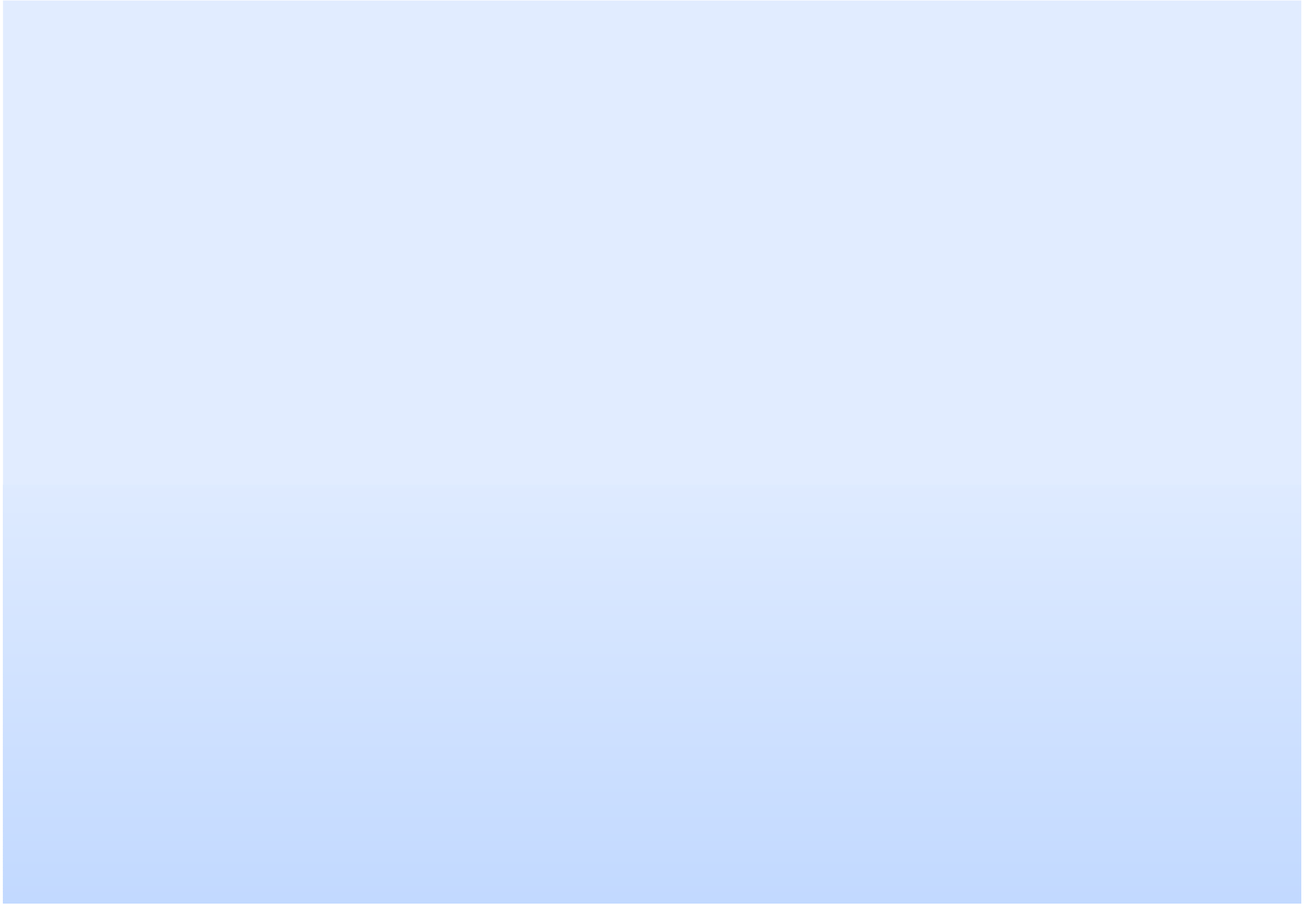
Non-intersecting random paths

- ▲ Given a 1-D diffusion process with transition probability density y .
- Suppose independent copies $- 1$ $-$ conditioned to
 - ▲ start at time t_0 at given points x_1, x_2, \dots ,
 - ▲ end at time t_1 at given points y_1, y_2, \dots ,
 - ▲ not intersect in the full time interval $[t_0, t_1]$.
- ▲ Then the positions of the paths at given time t are a determinantal point process. consequence of [Karlin-McGregor \(1959\)](#) theorem
- ▲ The correlation kernel takes the form

$$k(x, y) = \sum_{i=1}^n \phi_i(x) \psi_i(y)$$

where biorthogonal functions





Correlation kernel

- ▲ The correlation kernel for non-intersecting Brownian paths with two endpoints can be expressed in terms of the solution of the RH problem

$$y \frac{v}{-y} \begin{pmatrix} 1 & y \\ 2 & y \end{pmatrix} + y^{-1} + \begin{pmatrix} v \\ \end{pmatrix}$$

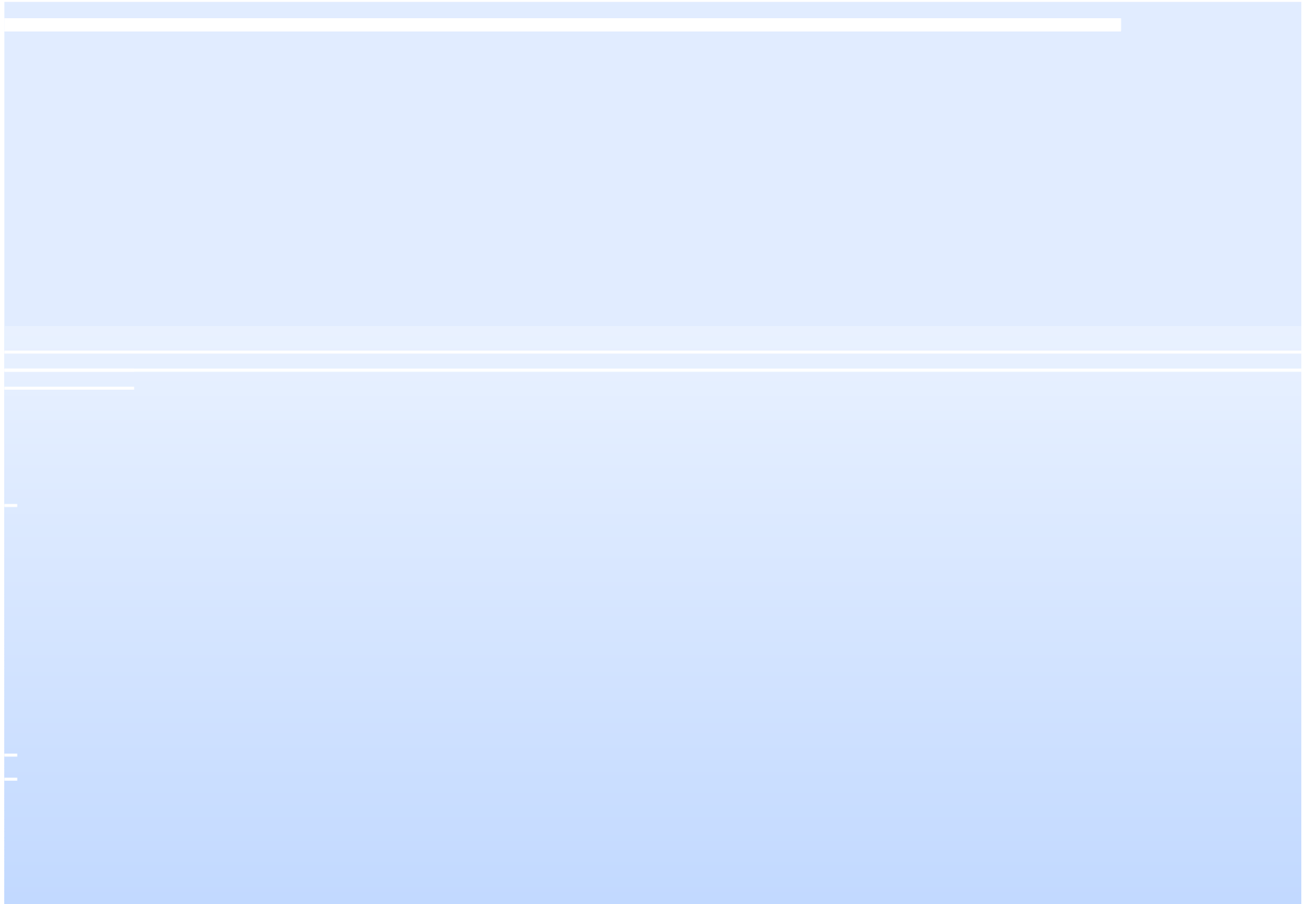
with weights $w_1 = e^{-x}$, $w_2 = e^{-x}$ where

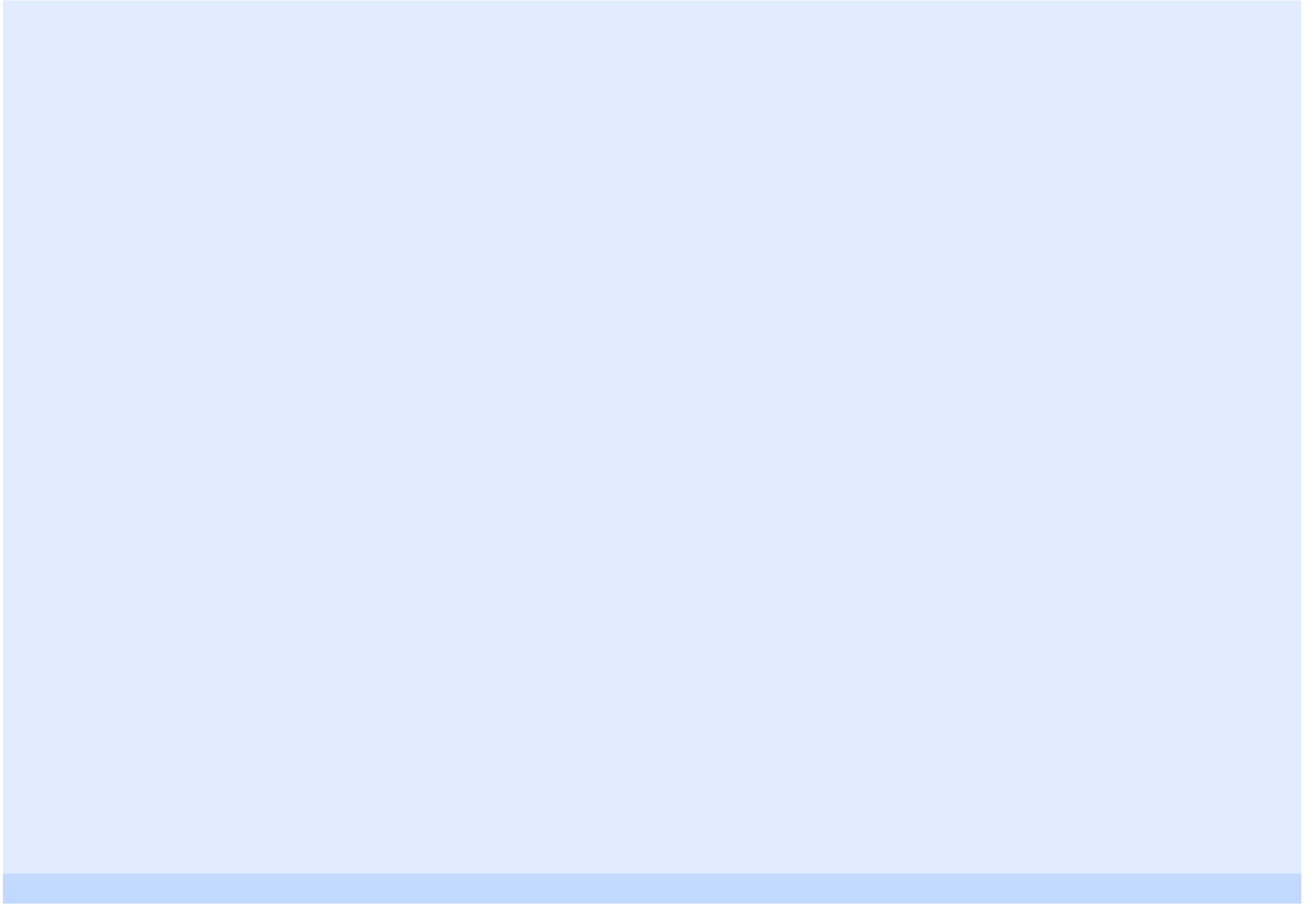
depends on

for two weights: [Bleher-Kuijlaars \(2004\)](#)

extension to more than two weights: [Daems-Kuijlaars \(2004\)](#)

- ▲ The RH problem is then analyzed with the steepest descent method for RH problems of Deift and Zhou.





Non-intersecting squared Bessel paths

▲ **PROPOSITION:**

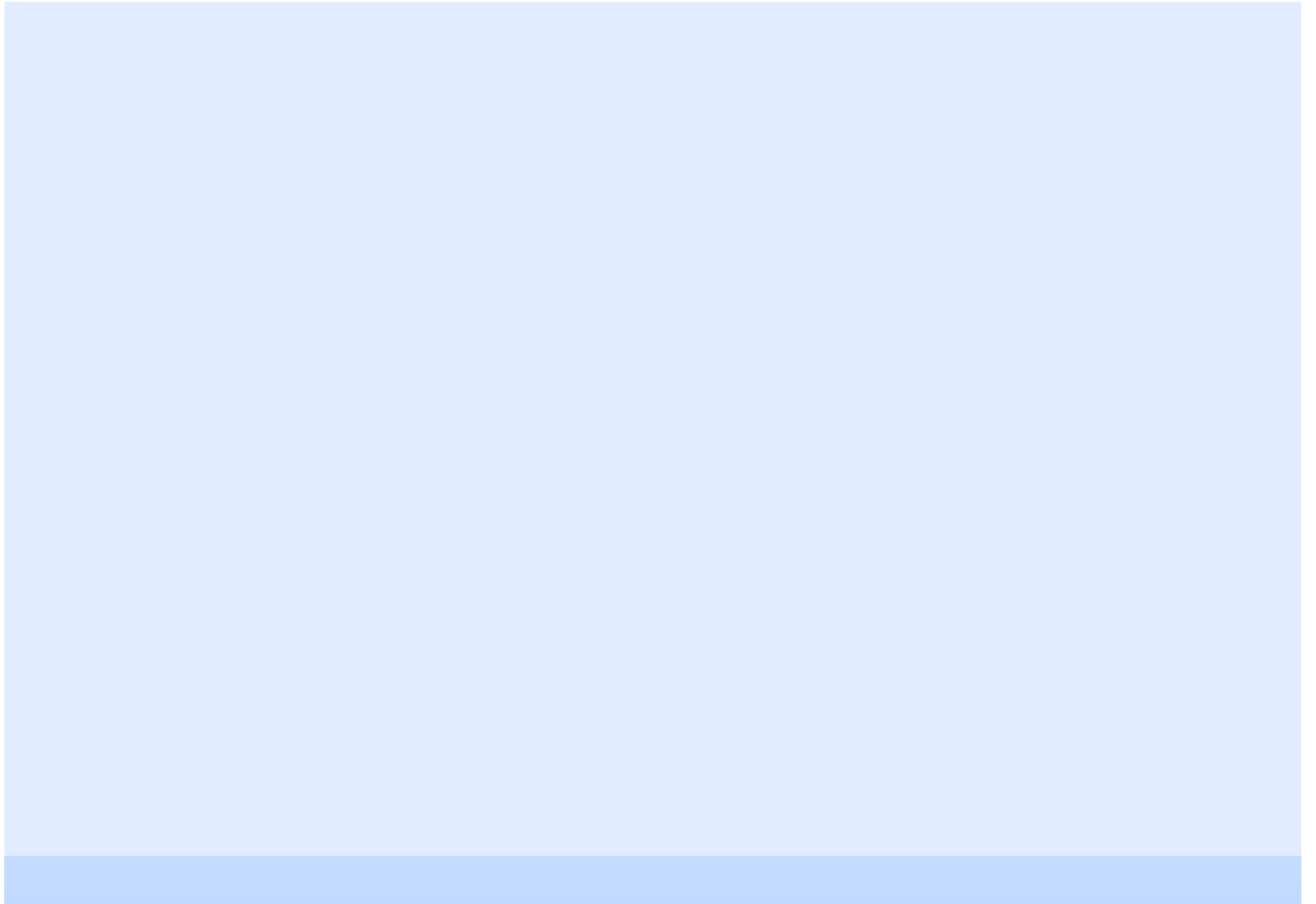
In the confluent limit $\epsilon \rightarrow 0$, $\lambda \rightarrow \lambda_0$, the positions of



Global regime 1

THEOREM on global regime

▲ As



THEOREM on local regime

▲ The correlation kernel $\kappa_n(x, y)$ has scaling limits as

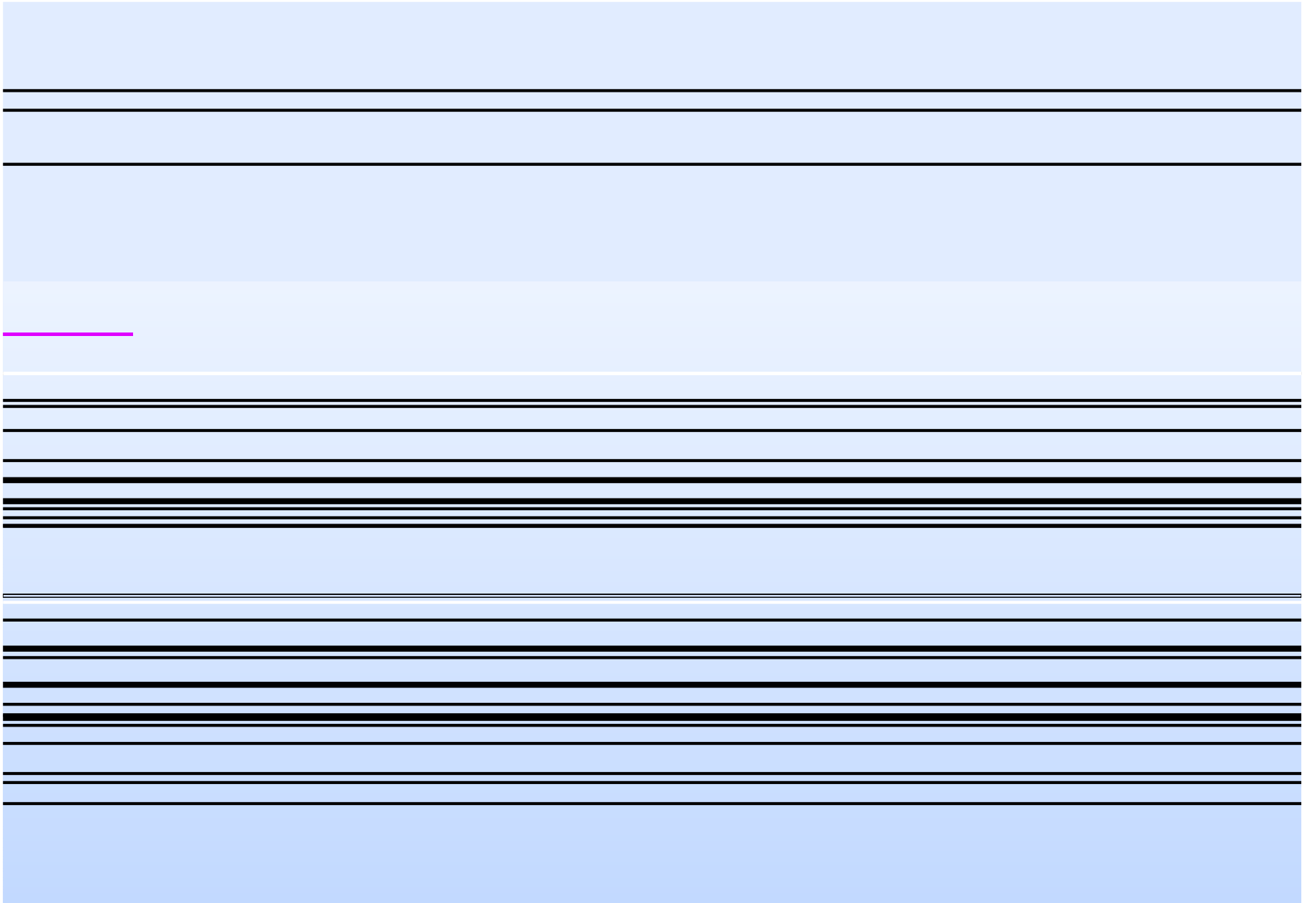
▲ Bulk scaling near a point x :

$$\frac{\text{S n} - y}{-y}$$

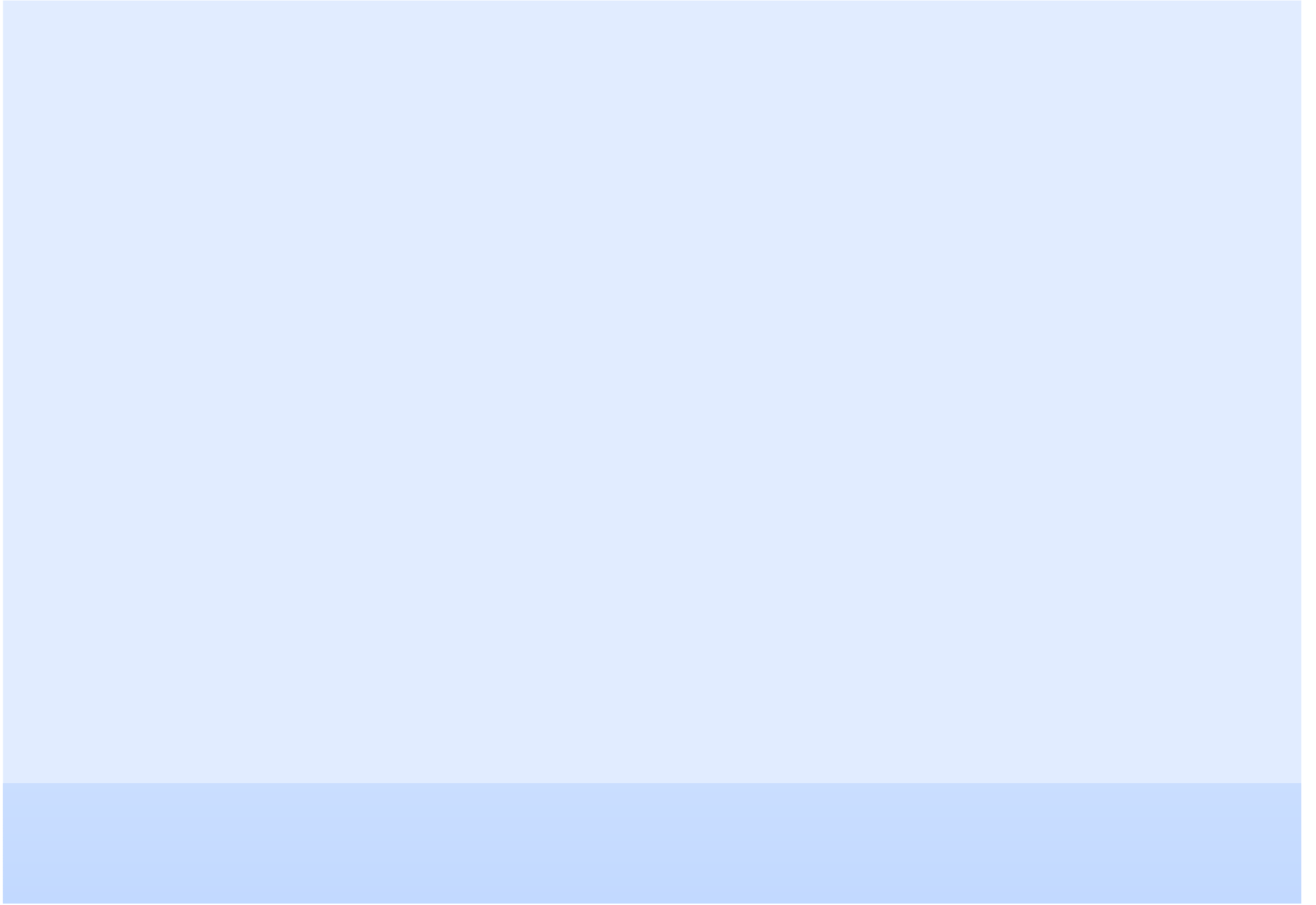
▲ Soft edge scaling near a point x or y :

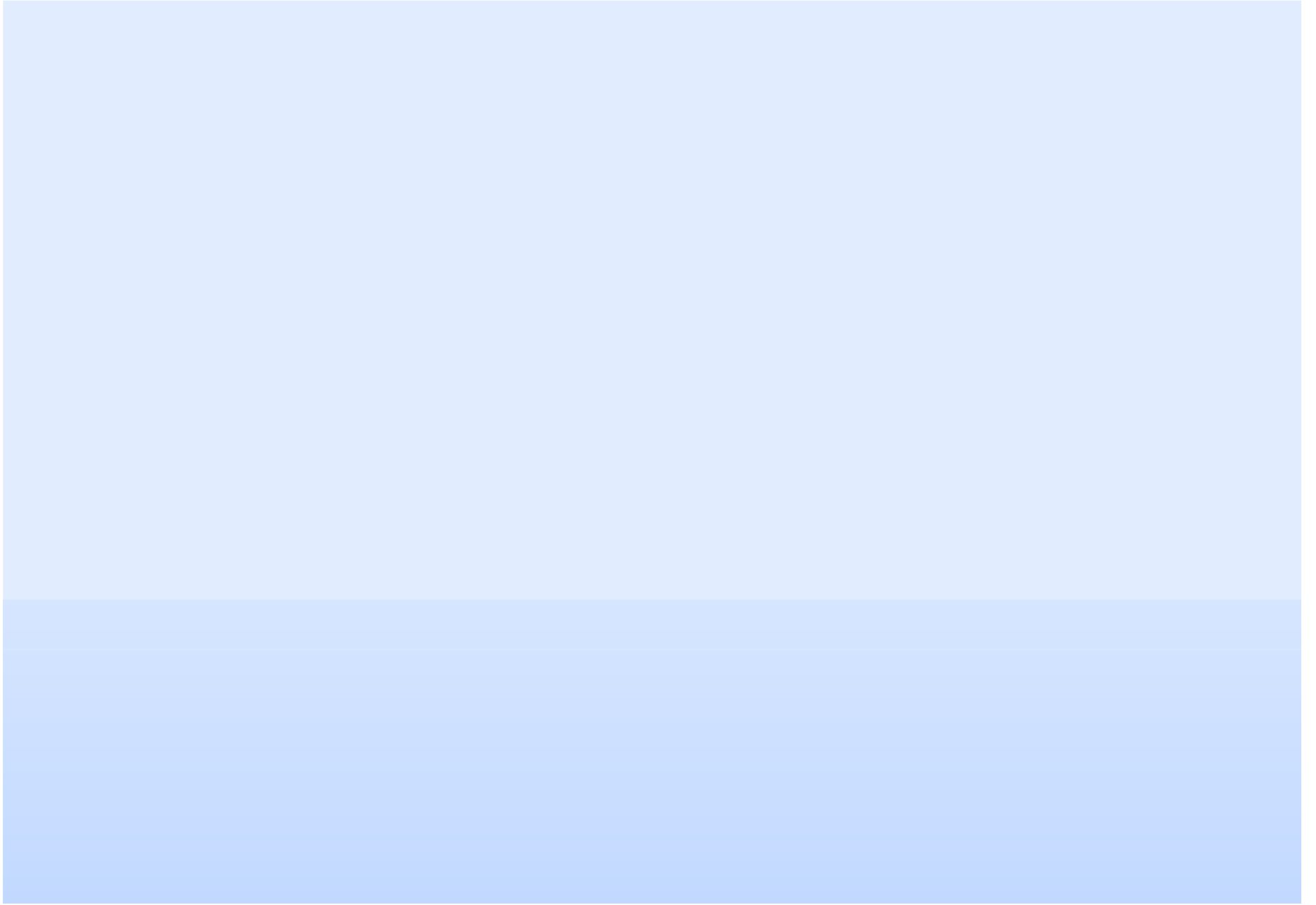
$$\frac{A - A y - A y}{-y}$$

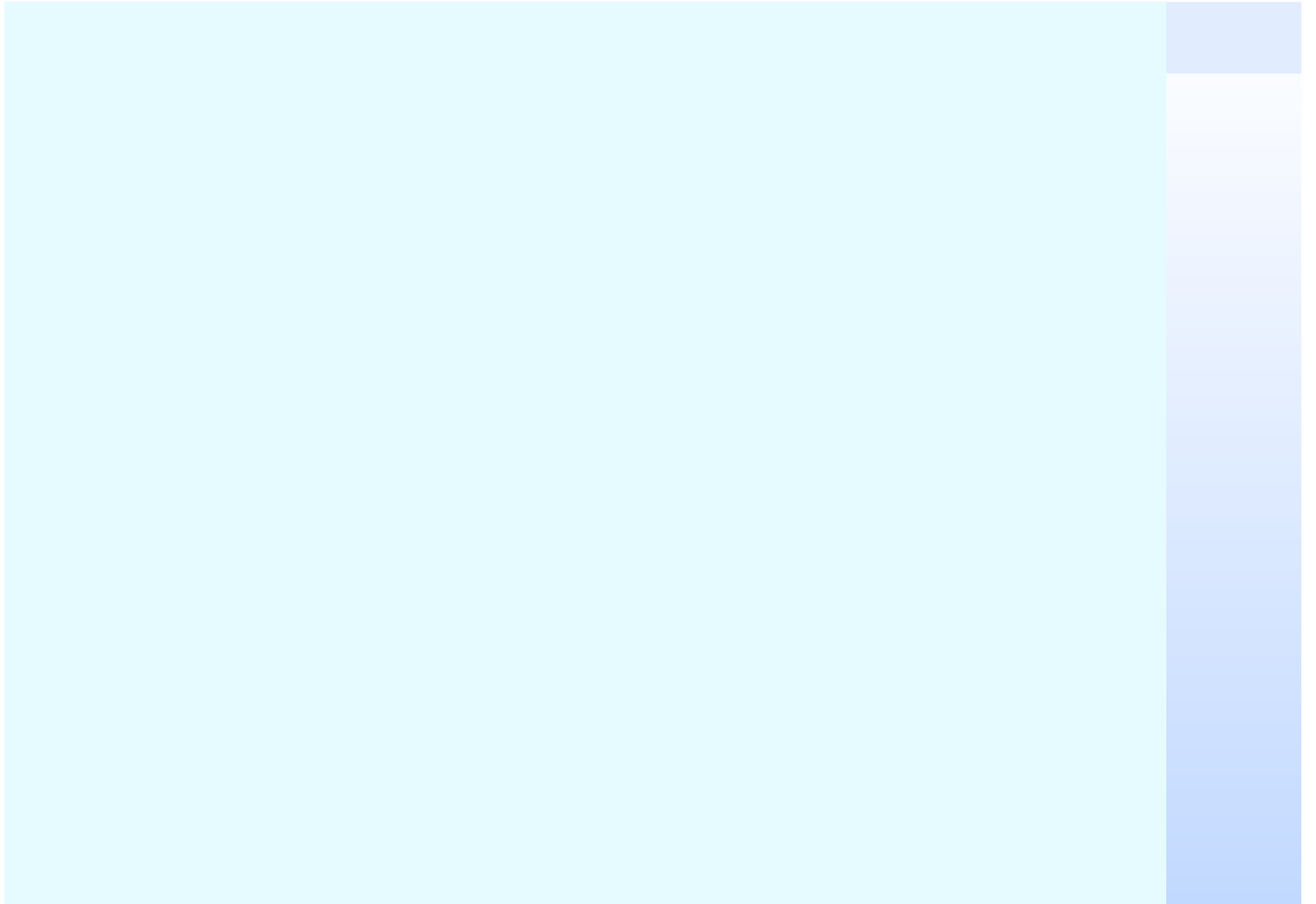
▲ Hard edge scaling near x if











Second transformation $X \quad U$

- ▲ Define the ρ -functions $\int \rho \log -$ and for certain constant matrices and ,

$$d \quad g \left(e^{- \quad (\quad) } e \left(- \quad \right) (\quad) e \quad (\quad) \right),$$

- ▲ Then we have jump matrices of the form

$$\begin{pmatrix} e^{-n} \quad \Delta x \quad x & 0 \\ 0 & e^{-n} \quad -\Delta x & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad x \in \Delta_1$$

(

