One-parameter deformation of Wishart – Laguerre ensemble

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Motivations

- Empirical or experimental data (e.g. in finance or QCD) can be modelled by positive definite random matrices (covariance or chiral models).
- Often, their spectral properties follow a power law distribution (time series of assets – instantons and delocalization in QCD etc…).
- Few solvable model so far for positive definite random matrices with fat tails!
- A deformed Wishart-Laguerre ensemble (exactly solvable!) can serve the purpose….

Power Laws and Random Matrices

- P

Wishart – Laguerre ensemble

$$P(\lambda_1,\ldots,\lambda_N) \propto \prod_{i=1}^{N} e^{-a\lambda_i} \lambda_i^{\nu} \prod_{j<k} |\lambda_j - \lambda_k|^2$$

One-parameter deformation

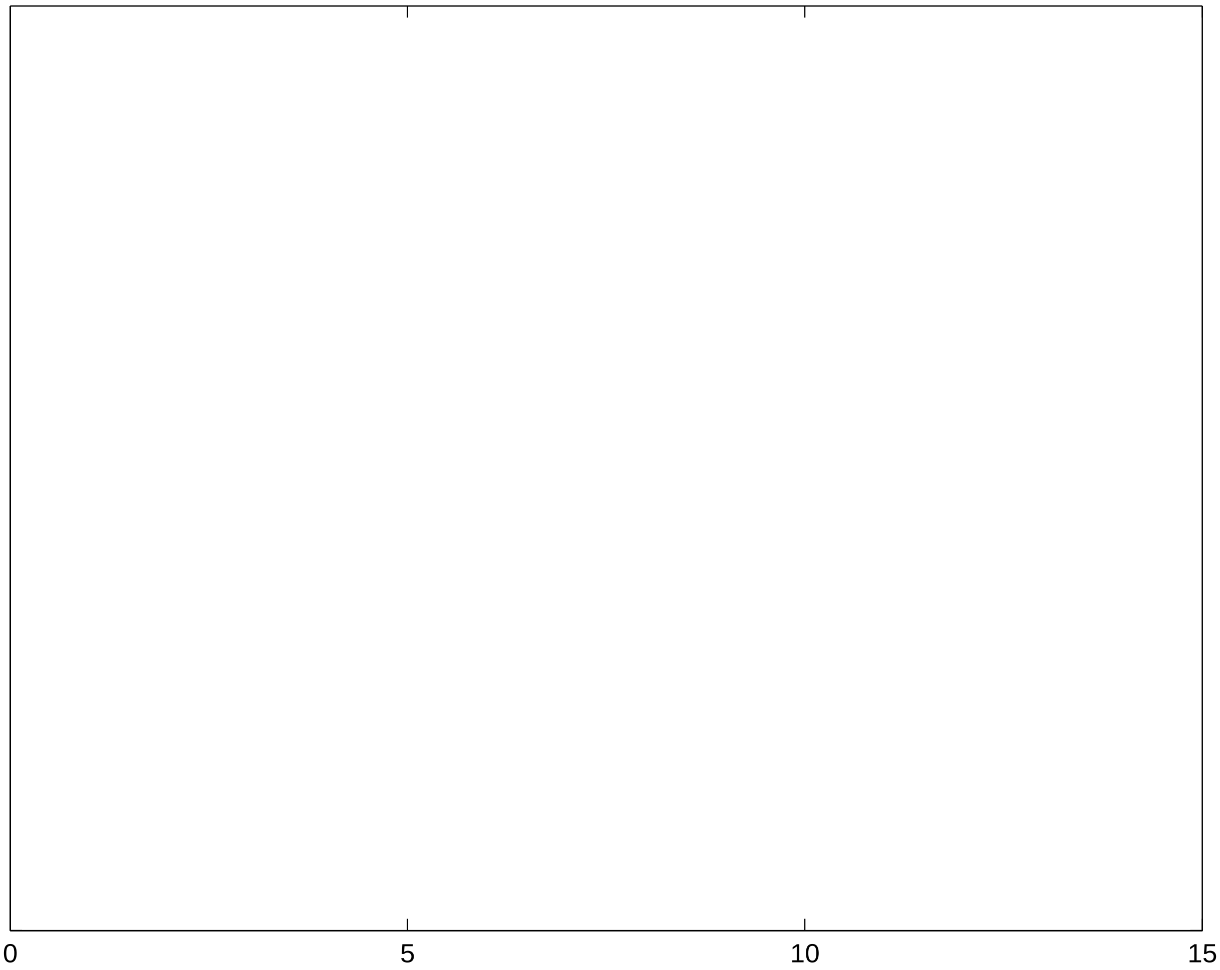
$$P_{\gamma;(N,\nu)}(\lambda_1,\ldots,\lambda_N) \propto \left(1+\frac{a}{\gamma}\sum_{i=1}^{N}\lambda_i\right)^{-\gamma} \prod_{i=1}^{N} \lambda_i^{\nu} \prod_{j<k} |\lambda_j - \lambda_k|^2$$

$\gamma \to \infty$ Back to Wishart - Laguerre

Gamma-transform Representation

$$\left(1+\frac{a}{\gamma}\mathrm{Tr}\,X\right)^{-\gamma}=\frac{1}{\Gamma(\gamma)}\int_0^\infty d\xi\, e^{-\xi}\xi^{\gamma-1}\exp\left(-\frac{a\xi}{\gamma}\mathrm{Tr}\,X\right)$$

Smallest Eigenvalue (Finite N)



Smallest Eigenvalue (Finite N)

ν positive integer for Wishart-Laguerre Unitary Ensemble: results by Dumitriu in terms of hypergeometric functions of a matrix argument.

For ν=0, simpler expression available

$$\text{Prob}[\, x < \lambda^{(\nu=0)}_{\min,N} < x + dx] = aN \, \exp\left(-aNx\right)$$

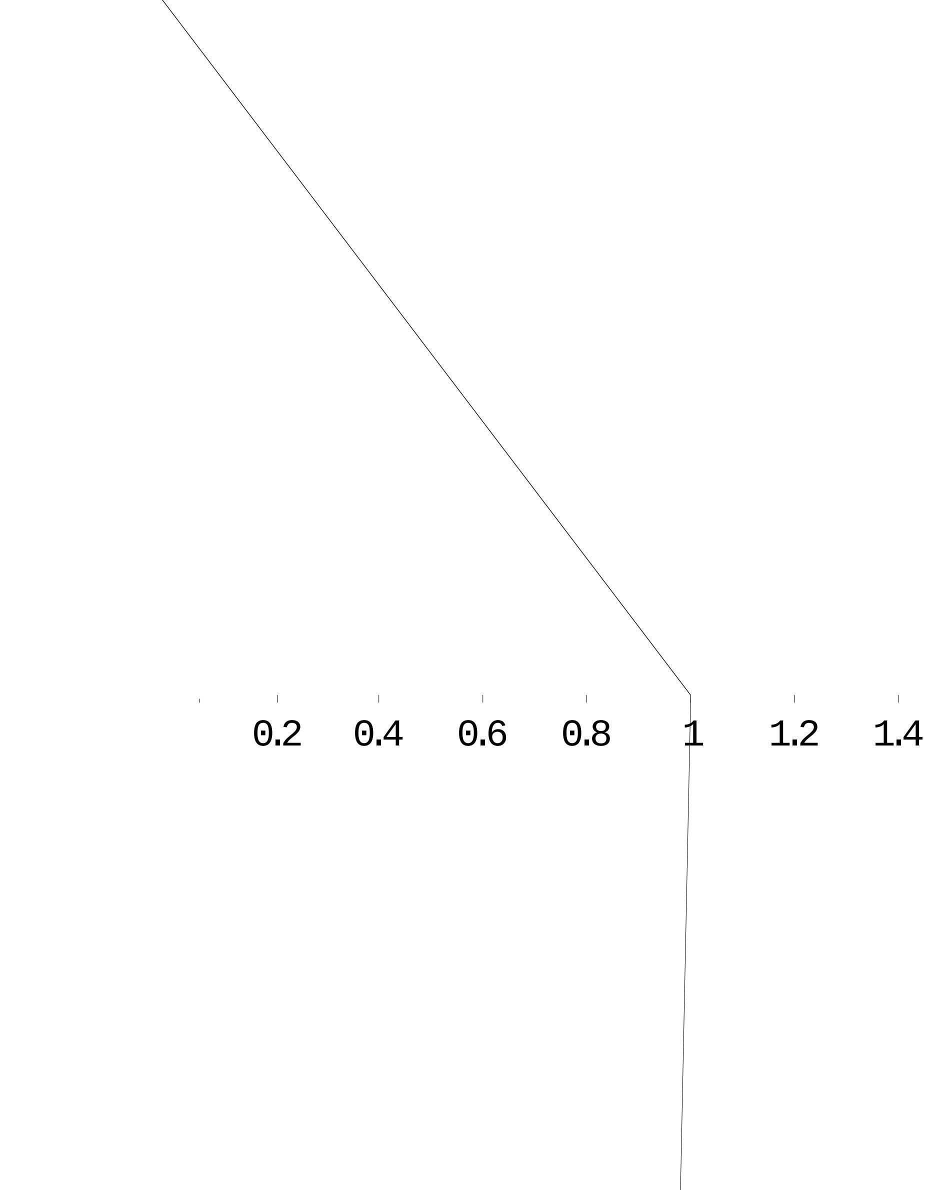
which gets modified in the deformed case as:

$$\text{Prob}[x < \lambda^{(\nu=0)}_{\min,N,\gamma} < x + dx] = \frac{aN(\gamma-1)}{\gamma}\left(1+\frac{aN}{\gamma}x\right)^{-\gamma}$$

Spectral Density (Macroscopic)

- Modification of the large N limit due to the constraint
- One can take

Spectral Density (Macroscopic)



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