Superbosonization

(a new effective-field method for random matrix and disordered electron systems with local gauge symmetries)

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Symmetric spaces

Riemannian symmetric superspaces: definition

Consider sections of (F_x) M (superfunctions),

M with values (x) (F_x)

Example

$$_1 \quad U \quad / (U \quad U)$$

Example (continued)

V_0 p	0	0	1	1
V_0 q	0	0	1	1
V_1 r	1	1	0	0
V_1 s	1	1	0	0

Lie superalgebra $(\ {}_0 \quad {}_0)\ (\ {}_1 \quad {}_1)$

$(\ {}^{p\ q} \,|\, {}^{r\ s})$ $\quad {}_{p\ q|r\ s}$

Important:

$$\left\langle e^{\mathrm{i}\,\mathrm{Tr}\,HK} \right\rangle_{\mathrm{GOE}} \qquad e^{\mathrm{i}\,\mathrm{Tr}\,HK} \qquad (\quad) \qquad e^{\;(b^2/2N)\,\mathrm{Tr}\,K^2}$$

Background & motivation (continued)

Pruisken-Schäfer domain

$Rv \quad v$: $v^t s v \quad 0$ space-like, $v^t s v \quad 0$ time-like, **0**

Every $O_{p,q}$ diagonalizable matrix R has p space like and q time like eigenvalues.

Encode ordering by motif, e.g., (R) **0 00** $(p \quad q \quad 3)$.
Associate with each motif a domain D by closure.

Pruisken Schäfer domain D **U** D is a union of $\begin{matrix} p & q \\ p & \end{matrix}$ $\begin{matrix} p & q \\ & q \end{matrix}$ domains. Each D is $O_{p,q}$ invariant.
$D \quad D$ for has co dimension 2.

Statement of result

Let $|dR| \qquad {}_{i \quad j} dR_{ij}$ (Lebesgue measure)

Theorem (FWZ). There exists some choice of cutoff function

$(\quad) \, (\qquad\qquad\qquad 0)$

$\mathrm{sgn}(\quad) \qquad 1 \qquad\qquad C \; ,$

Corollary

Formulation in terms of eigenvalues: Let $R \quad g \quad g^{1}$ with $\mathrm{diag}(\;_{1},\ldots,\;_{p\;q})$ and $g \quad SO_{p,q}$. Volume element

$|dR| \quad J(\;)\,|d\;|\;|dg|$ where $|d\;| \quad {}_{i\;1}^{p\;q} d\;_{i}$ and dg Haar

$SO_{p,q}$. J cs 0 0.35294 0.78431 scn 184.26 73.431 10.38 11.22 ref $J(\;) \; |\;_{i}\;_{j}|$

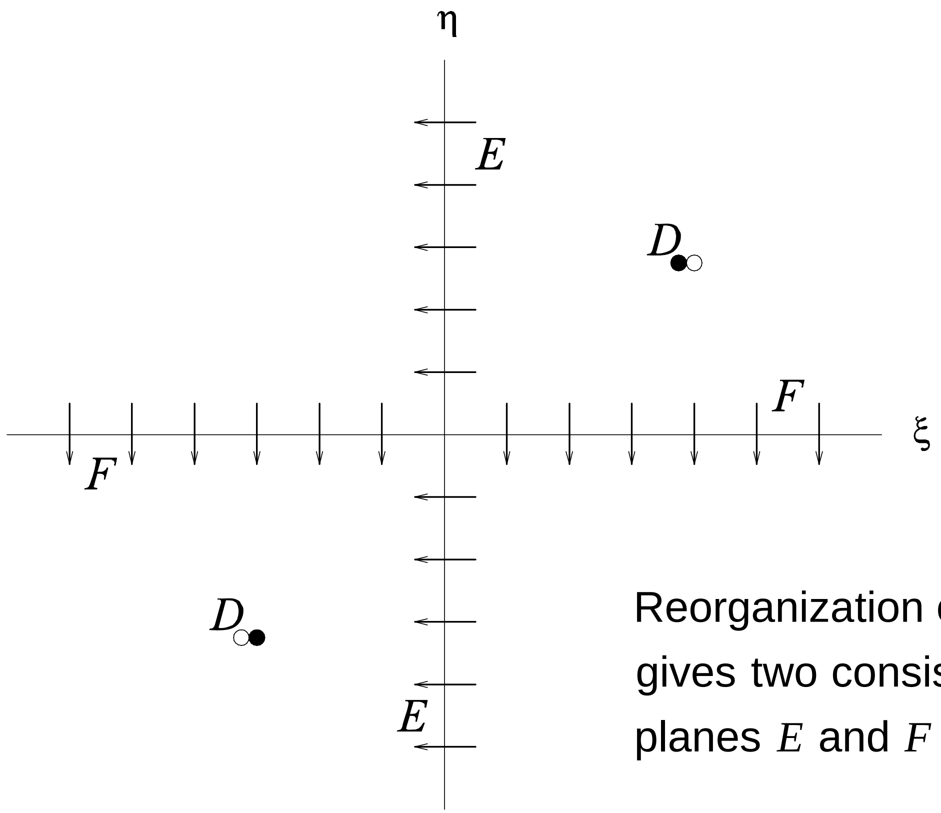
Two domains for p = q = 1

$R \quad r_{11} \quad r_{12}$

$r \quad r$

Light-cone coordinates:

Reorganization of boundary components



Reorganization of the boundary pieces gives two consistently oriented boundary planes E and F.

Superbosonization

- P. Littelmann, H.-J. Sommers, M.R.Z., Commun. Math. Phys. (in press)
- J.E. Bunder, K.B. Efetov, V.E. Kravtsov, O.M. Yevtushenko, M.R.Z., J. Stat. Phys. 129 (2007) 809

Motivation: supersymmetry method

Central object of the theory is the characteristic function of the

on, (K) $\mathrm{e}^{\mathrm{i}\,\mathrm{Tr}\,HK} d$ (H), where $(Z$ commuting,

anticommuting variables) K p Z $\tilde{Z}$ q $\tilde{}$

Special case: commuting variables only

Let $p \quad 1, q \quad 0$ and consider GL_N invariant holomorphic function $f : \ ^N \quad (\ ^N) \qquad , \ f(z, \tilde{z}) \quad f(gz, \tilde{z}g \ ^1), \ g \quad \mathrm{GL}_N$.

Fact (from invariant theory) : there exists a holomorphic function $F : \qquad$ such that $F(\tilde{z} \ \ z) \quad f(z, \tilde{z})$.

By push forward of the integral one has

$$_N \ f(z, z \ \ ) d^{2N} z \quad c_N \quad F(r) r^{N \ \ 1} dr$$ (if the integral exists).

generalization: see Fyodorov, Nucl. Phys. B **621** (2002) 643

Special case: Grassmann variables only

Let $F:$ be a holomorphic function. For a vector

$_1, \ldots,$ $(\overline{} \quad).$

$(\overline{} \quad) \; \overline{} \quad :$

Symmetry argument (heuristic)

Pullback $f \quad F$. Compare two distributions:

Distribution 1: ${}_1[F] \quad {}_{\tilde{Z} \;\; Z} \quad F$

Distribution 2: ${}_2[F] \quad {}_M DQ\, \mathrm{SDet}^{N/2}(Q)\, F(Q)$

For $g \quad \mathrm{OSp}_{2p|2q}$ let $F_g(Q) : \quad F(gQg^T)$.

The transformation behavior is the same:

$${}_A[F_g] \quad \mathrm{SDet}^{\,N}(g) \quad {}_A[F] \qquad (A \quad 1, 2).$$

Comments

Application: Wegner's N-orbital model, U(N)

Hilbert space $V \quad {}_{i} \quad V_i$, orthogonal projector $\quad {}_{i} : V \quad V_i$

$$: \quad e^{\mathrm{i}\,\mathrm{Tr}\,HK} d \quad (H) \quad e^{\;(1/2N) \quad {}_{i,j} C_{ij}\, \mathrm{Tr}\, K \quad {}_{i} K \quad {}_{j}}$$