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A Random Matrix Model for the Doorway Mechanism

Overview

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- Motivation
  Giant resonances and Superscars

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- In.07877095 S4]TJ ET Q.s08o1- 1 0 0 rg-0.084544141788(o)-0.079

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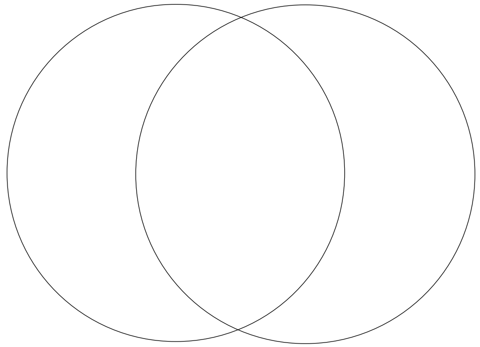
- Conclusions

1. Example: Electric Giant Dipole Resonance

seen in many nuclei, here for Gold (**A** 197)



Strength Function

cross section contains huge number of individual states (fragmentation) which cannot be resolved

strength function is related to this: consider state $|\mathrm{GDR}\rangle$ with resonance energy E_{GDR} and couple many states to it
→ density around $|\mathrm{GDR}\rangle$

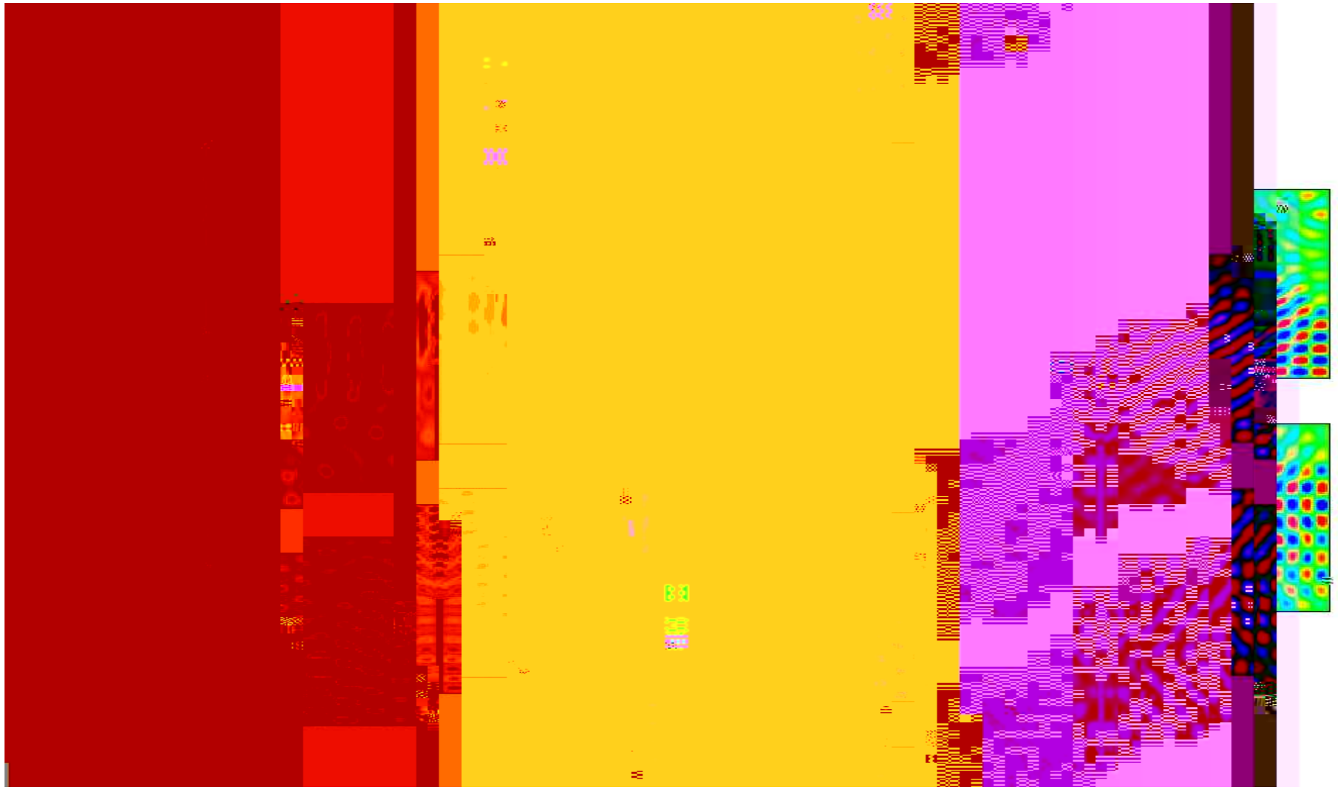
$$\rho_{\mathrm{GDR}}(E) = \frac{1}{\pi}\,\frac{\Gamma/2}{(E - E_{\mathrm{GDR}})^2 + \Gamma^2/4} \qquad \text{(Breit–Wigner)}$$

under rather general conditions!

⟶ strictly, one cannot conclude chaotic fluctuations,
but at these GDR energies one certainly expects them

Bohr, Mottelson, Nuclear Structure (1969); Zelevinsky, Annu Rev Nucl Part Sci 46 (1996) 237

Superscars in a Pseudointegrable Barrier Billiard



ordinary scars “vanish” at high energies, superscars do not !

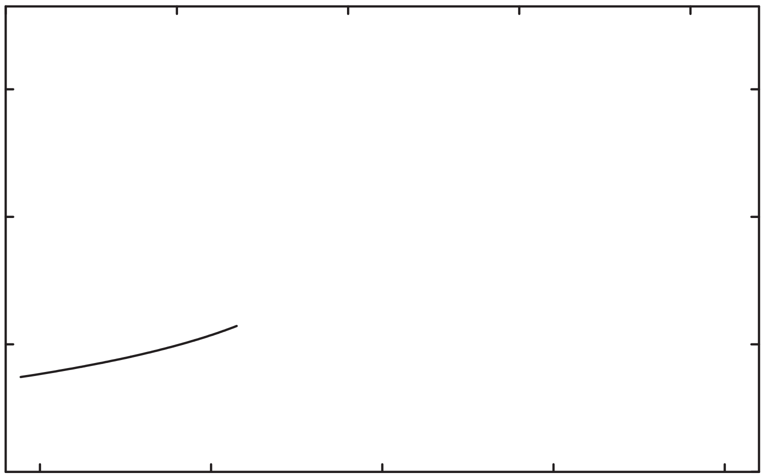
Bogomolny, Schmit, PRL 92 (2004) 244102

Bogomolny, Dietz, Friedrich, Miski–Oglu, Richter, Schäfer, Schmit, PRL 97 (2006) 254102

Local Density of States

sum over measured states **λ** and over **m** quantum number

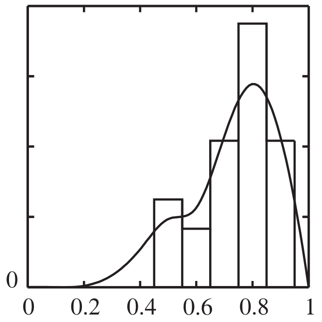
$$\rho_n(\tilde{f}) \quad \left\langle \sum_{\lambda} |c_{m,n}(\tilde{f})|^2 \delta\left(\tilde{f} - \tilde{f}_{\lambda} + \tilde{f}_{m,n}\right) \right\rangle_m$$



Maximal coupling strength versus Breit Wigner width

distribution of squared maximum coefficient $c^2_{\max}$ [illegible] $(c^2_{m,n})$

distribution of all coefficients $c^2_{m,n}$



Spectral observables

- Distribution $\mathbf{P}_{\max}$ of the maximal overlap with the Doorway state

$$\mathbf{P}_{\max}(|\mathbf{c}_{\max}|) = \left\langle \boldsymbol{\delta} \left(|\mathbf{c}_{\max}| - \max \right. \right.$$

Task: Average over random Hamiltonian H_B and couplings

Overcome the problem

- Fourier transform: $\mathbf{Q}(\mathbf{u}) \quad \frac{1}{2\pi}\int \mathbf{dk}\,\mathbf{e}^{\mathbf{ik}(\mathbf{u}-1)}\mathbf{R}(\mathbf{k})$

$$\mathbf{R}(\mathbf{k}) \qquad \det(\mathbf{H}_{\mathbf{B}}^2 + 2\mathbf{iv}^2\mathbf{k})^{-\beta/2}|\det \mathbf{H}_{\mathbf{B}}|^{\beta}\rangle_{\mathbf{N}}$$

- Crucial

$$\left\langle \det(\mathbf{H}_B + 2\mathbf{iv}\;\;\mathbf{k})^{-\;\;/}\;|\det \mathbf{H}_B|\;\;\right\rangle_N \;\square\; \lim_{\to 0}(\;+2\mathbf{iv}\;\;\mathbf{k})\;\;/\;\left\langle \mathrm{tr}\,\boldsymbol{\delta}(\mathbf{H})\det(\mathbf{H}\;\;+2\mathbf{iv}\;\;\mathbf{k})^{-\;\;/}\right\rangle_{N+1}$$

- Undo Fourier transform

$$\mathbf{Q}(\mathbf{u}) \;\square\; \frac{\mathbf{d}^{\beta/2}}{\mathbf{du}^{\beta/2}}\int_0^\infty \mathbf{dx}\,\mathbf{x}^{\beta(\mathbf{N}+1)/2-1}\mathbf{F}_{\mathbf{N}+1}\left(\;\;+\frac{\mathbf{x}}{\mathbf{v}^2}\right)\boldsymbol{\delta}(\mathbf{u} - \quad - \mathbf{x})$$

$$\frac{\mathbf{d}^{\beta/2}}{\mathbf{dx}^{\beta/2}}\boldsymbol{\delta}(\mathbf{x}) \qquad \lim_{\to 0}\frac{1}{2\pi}\int \mathbf{dk}(\;+\mathbf{ik})^{\beta/2}\mathbf{e}^{\mathbf{ikx}}$$

Supersymmetric method

- The function

$$\mathbf{F}_{\mathbf{N}+1}(\mathbf{g}) \ \square \ \left\langle \mathrm{tr}\, \boldsymbol{\delta}(\mathbf{H}) \mathbf{e}^{-\frac{i}{2} \mathrm{tr}\, \mathbf{G}' \mathbf{H}} \right\rangle_{\mathbf{N}+1}, \quad \mathbf{G}' \quad \mathrm{d} \ \mathrm{g}(\mathbf{g} - \ , \ , \ldots, \ )$$

  is a generalised eigenvalue density in the band center

- Calculation of $\mathbf{F}_{\mathbf{N}+1}(\mathbf{g})$:
  - mapping onto a supersymmetric two matrix model
  - one matrix integral with sadddle–point approximation the other exact
- $\mathbf{F}_{\mathbf{N}+1}(\mathbf{g})$ the same for GOE and GUE

Results

GUE: $\alpha = \sqrt{\overline{V^2}}/d$ dimensionless coupling strength

$$P_0(|c_{0s}|) = \frac{2\pi\alpha^2}{(1-|c_{0s}|^2)^3} e^{-\frac{\pi\alpha^2|c_{0s}|^2}{(1-|c_{0s}|^2)}} \left(1 + \frac{4\pi^2\alpha^2|c_{0s}|^2}{1-|c_{0s}|^2}\right)$$

GOE: $\alpha = \sqrt{\overline{V^2}}/d$ dimensionless coupling strength

$$P_0(|c_{0s}|) = \frac{\pi^3\alpha^6|c_{0s}|^4}{2(1-|c_{0s}|^2)^5} e^{-\frac{\pi\alpha^2|c_{0s}|^2}{2(1-|c_{0s}|^2)}}$$

$$\left[K_0\left(\frac{\pi^2\alpha^2|c_{0s}|^2}{4(1-|c_{0s}|^2)}\right) + K_1\left(\frac{\pi^2\alpha^2|c_{0s}|^2}{4(1-|c_{0s}|^2)}\right)\right]$$

K_n: modified Bessel function

S. Åberg, T. Guhr, HK in preparation

Plot of results

P_{max} *versus* P_0 *(V / d α)*

Summary and Conclusions

- doorway mechanism in nuclei and chaotic systems
- superscars provide a beautiful model for doorway mechanism
- new observables:
  - $\mathbf{P}_{\mathrm{max}}$: Distribution of the maximal overlap of the doorway with the eigenstates
  - $\mathbf{P}_0$: Distribution of the overlap of the doorway state with itself
- Exact large **N** solution for $\mathbf{P}_0$.
- Interesting mathematics
  - Generalized Eigenvalue density
  - **β** dependence through fractional derivative
  - Supersymmetric two–matrix model