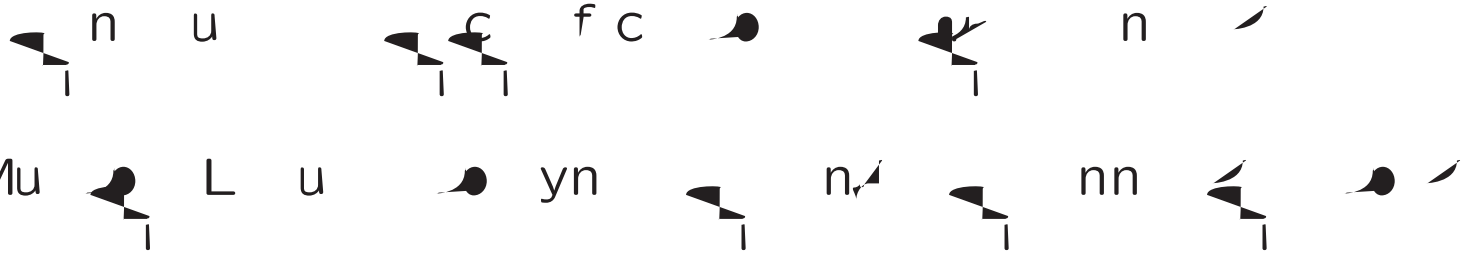
# Universality in complex Wishart ensembles

M. M X . , X .



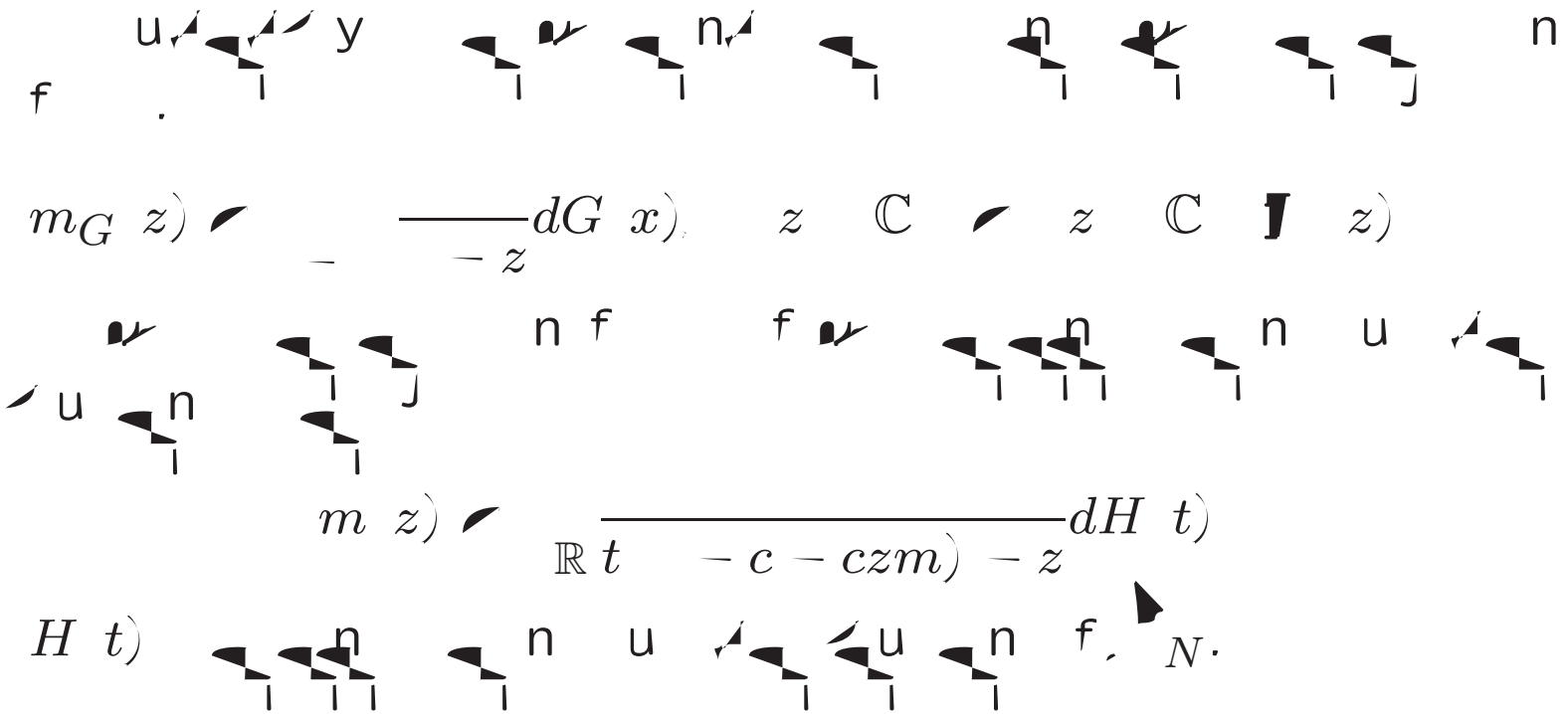
# Results

. $N$ nc n u , $N$ f n $N$ f $a$.

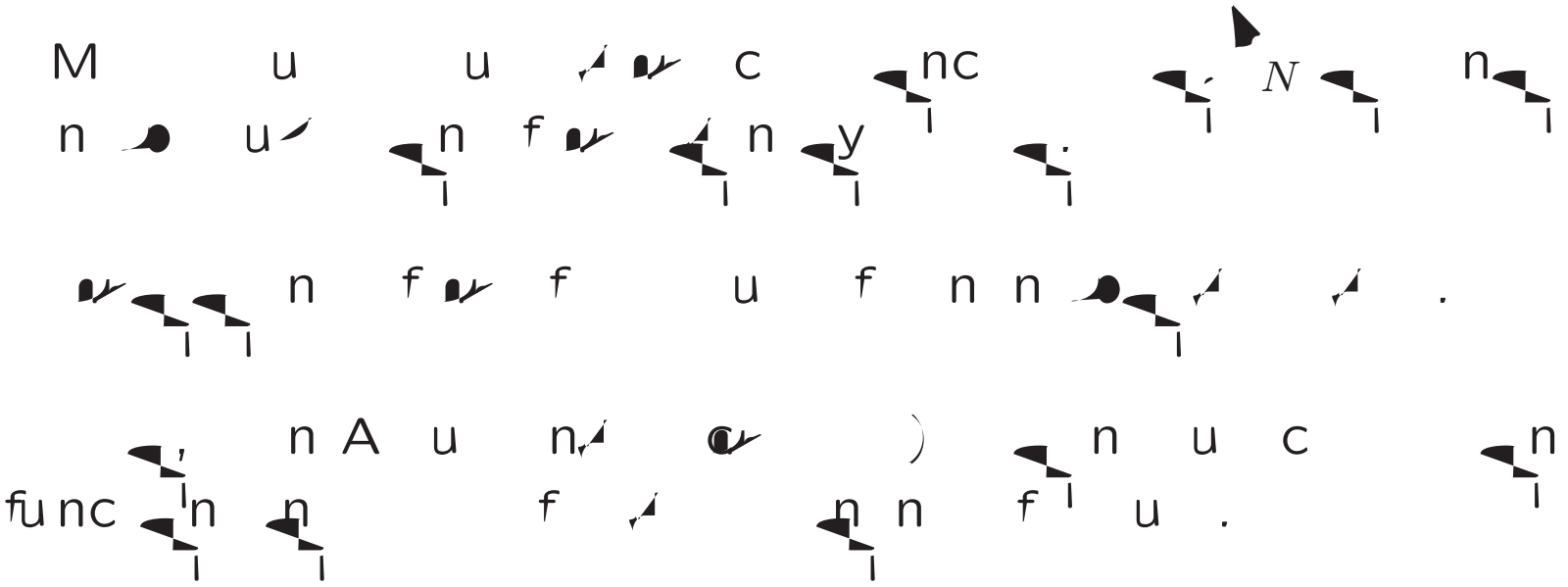
$M$ $N$ n $N \to$ , $N$ , $N \to$ n $\frac{N}{M} \to c$, $\frac{N}{N} \to$ .

n y n u c n n y n n n u f c u n A y n n

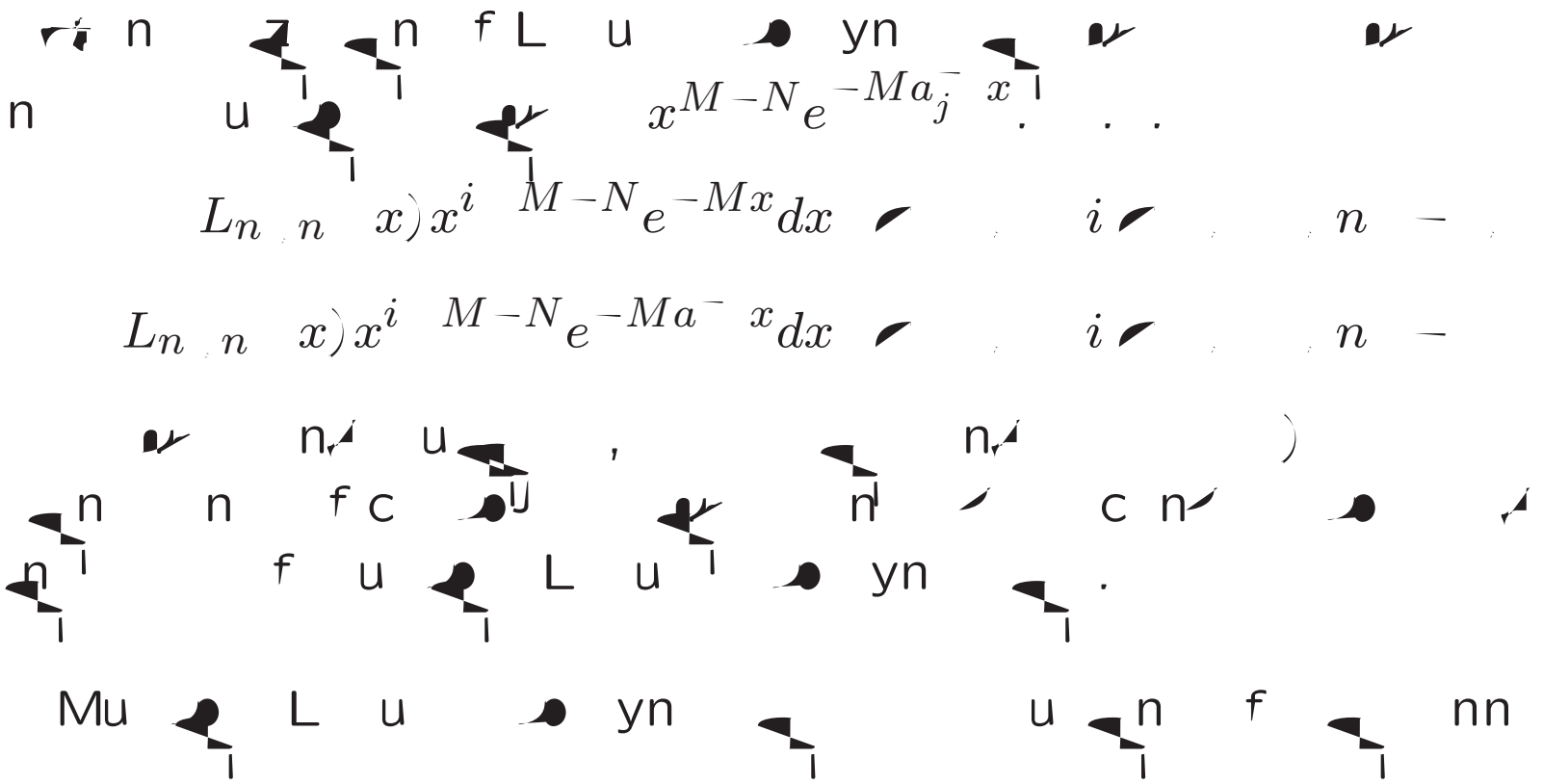
# Global eigenvalue statistics

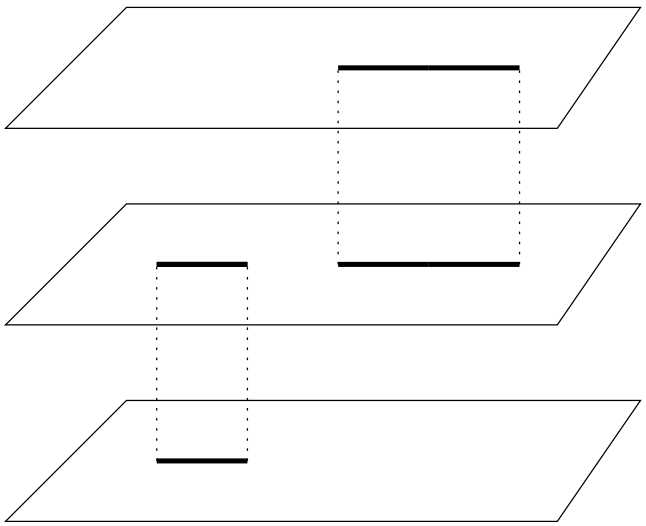


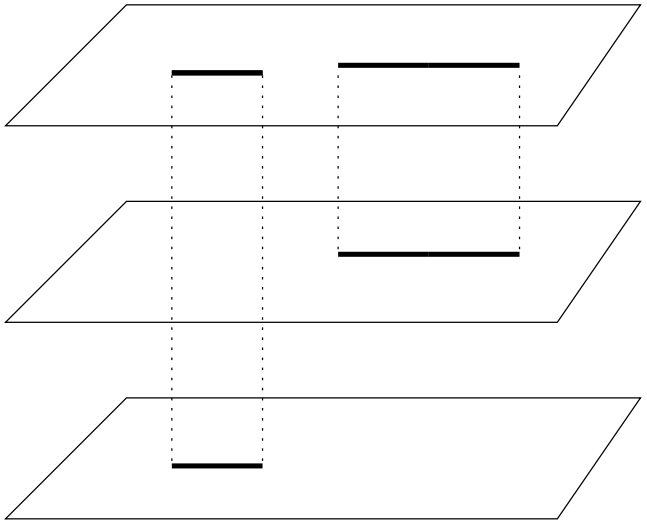
# Local eigenvalue statistics

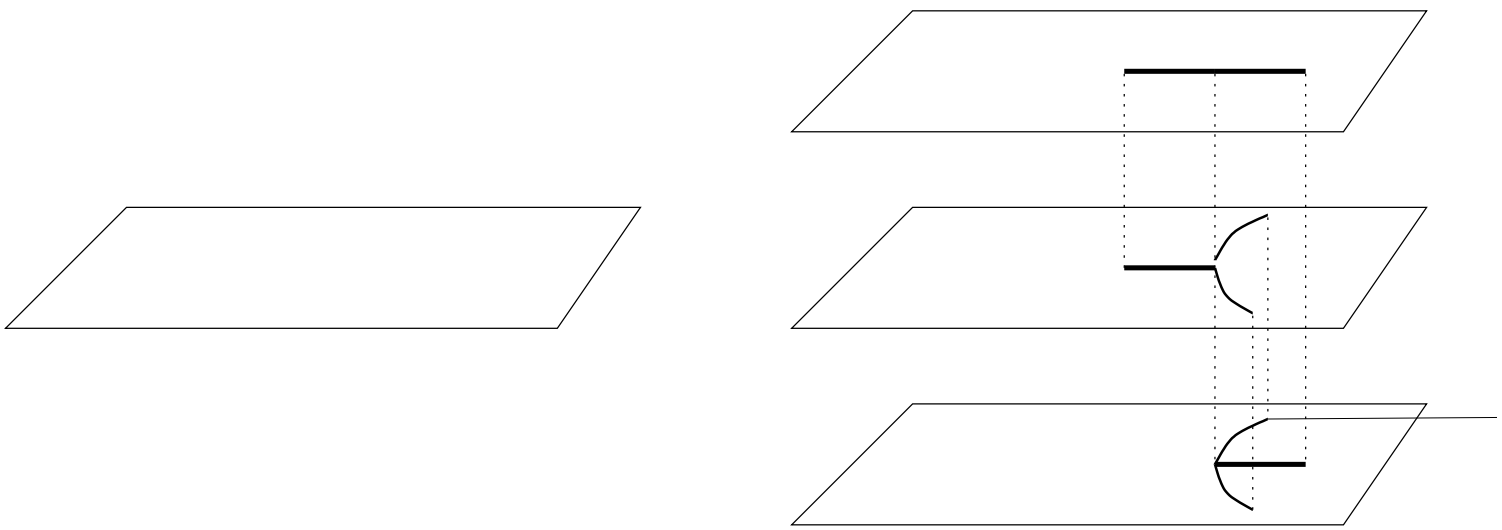


# Multiple Laguerre polynomials









# Stieltjes transform

n $\underline{F}$ y

$\underline{F}$ $-c)I$

n

. u n u n

$$z) \quad -\frac{}{z} \quad O\ z^{-}\ ), \qquad z \to$$

$$z) \quad - \quad \frac{c \quad - \ )}{z} \quad O\ z^{-}\ ), \qquad z \to$$

$$z) \quad -\frac{}{a} \quad \frac{c}{z} \quad O\ z^{-}\ ), \qquad z \to$$

. $m_{\underline{F}}$ u n . nc cu n

u f $\underline{F}$.

**Lemma 1 If** $z$ . u $\underline{F})$**, then** $m$ $m_{\underline{F}}$ $z)$ **satisfies the following.**

**1.** $m$ $\mathbb{R}$ **;**

**2.** $-\frac{}{m}$ . u $H)$**;**

**3.** $z'$ $m)$ **.**

**Conversely, if** $m$ **satisfies 1-3, then** $z$ $z$ $m)$ . u $\underline{F})$**.**

$z'$ n n f u .

Z f yn

$a \quad -c) \quad a \quad -c \;) \quad a \quad -c \quad - \;))$

$-c \quad - \;) \quad a \quad -c \;) \quad a) \quad a)$

c n n , , c n n

n ,

n f u . . n ,

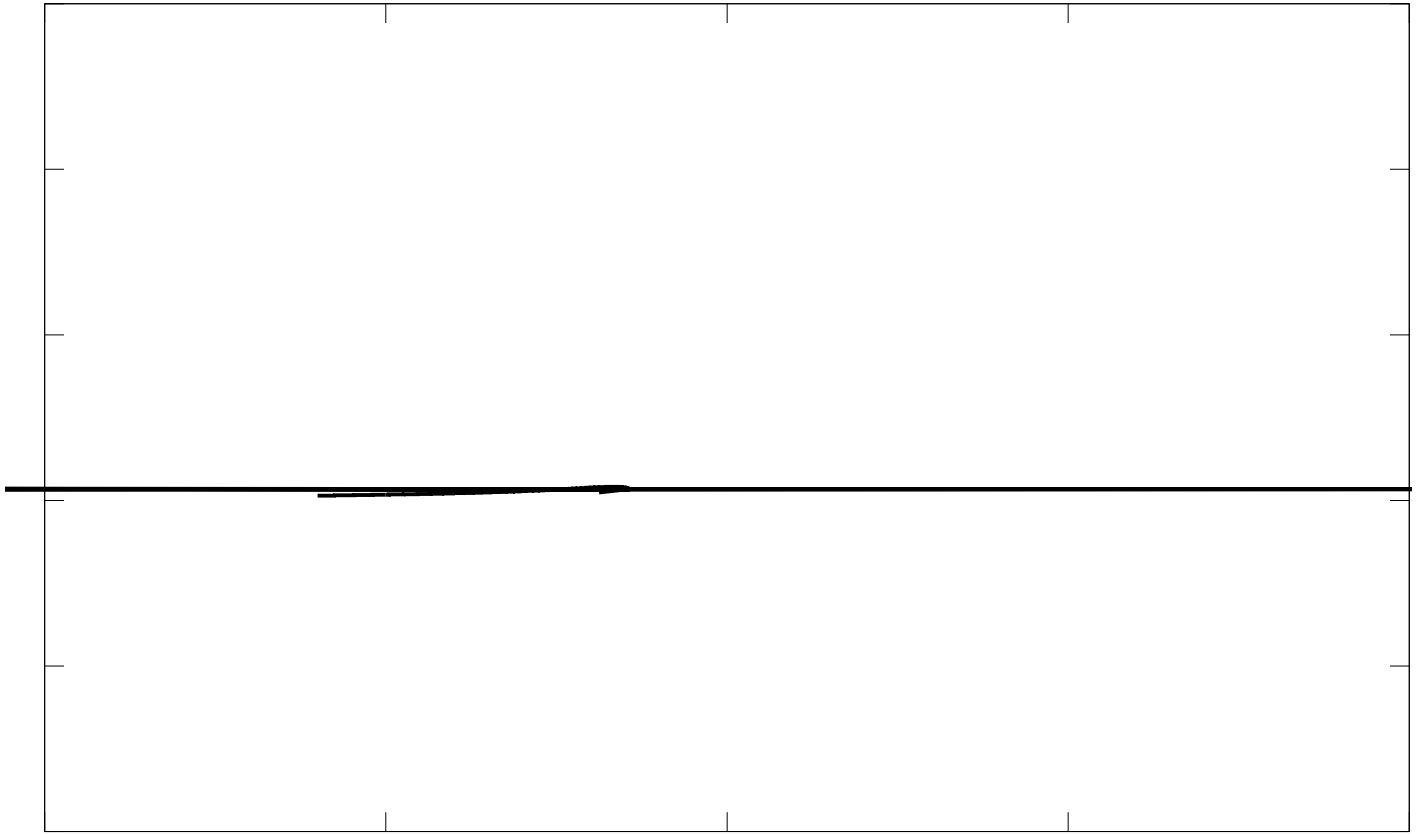
u $\underline{F})^c$ - ) ) ) )

$k$ n y

$$k \; -\frac{}{k} \quad c\frac{-}{k} \quad c\frac{a}{a \;\; k}$$

f , n u c n f

n , u n .



# Summary

u c n c nc
$N$ n u n $N$ n u $a$ n $\frac{N}{M} \to c$,
$\frac{N}{N} \to$ .

n f f c n u n f n n
.

n n n y n n n u n A y
n n . c y u n f n
u .

n f c cu n
c n f J nn n y .

n c nc n y ny nc n

u , n f nn u f c n

n y c n n z c n nn

u f c n u c n .