

Universality in complex Wishart ensembles

$M \times M$ X \cdot , X \cdot

n u

c f c

n

Mu L u

yn

n

n

nn

n

Results

N a. nc n u , N f n N f

M N n $N \rightarrow$ N , $N \rightarrow$ n $\frac{N}{M} \rightarrow c$,
 $\frac{N}{N} \rightarrow$.

n y n u c n n y n
 n n u f c u A y n n



Global eigenvalue statistics

$$\begin{aligned}
 & f \quad u \quad y \quad n \quad n \quad n \quad n \\
 & m_G(z) \quad \frac{dG(x)}{z} \quad z \in \mathbb{C} \quad z \in \mathbb{C} \quad \mathbf{I}(z) \\
 & u \quad n \quad n \quad f \quad f \quad n \quad n \quad u \\
 & m(z) \quad \frac{dH(t)}{\mathbb{R}t - c - czm) - z} \\
 & H(t) \quad n \quad n \quad u \quad u \quad n \quad f, N.
 \end{aligned}$$

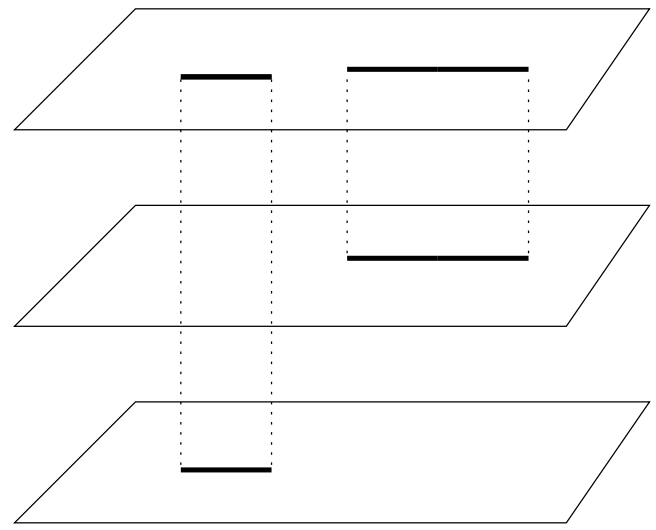
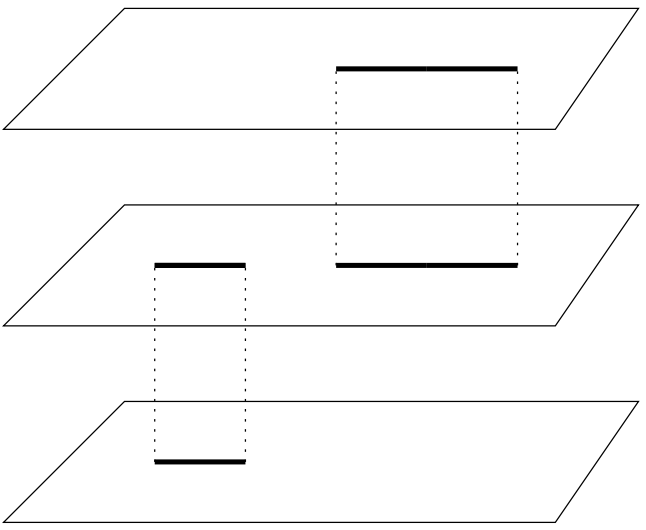
Local eigenvalue statistics

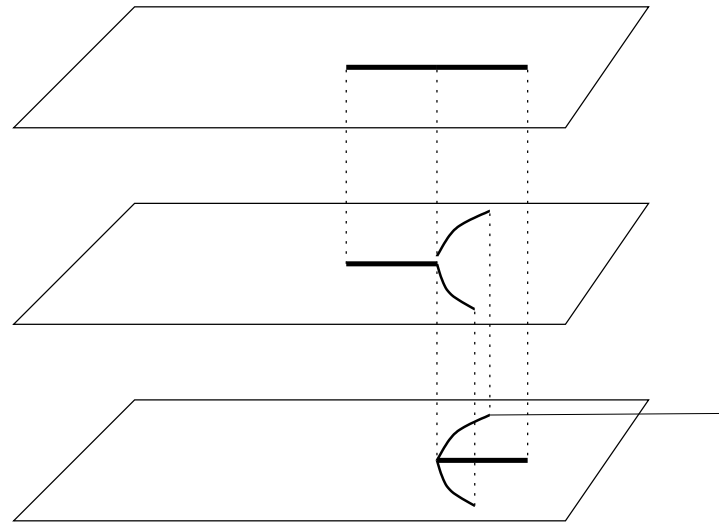
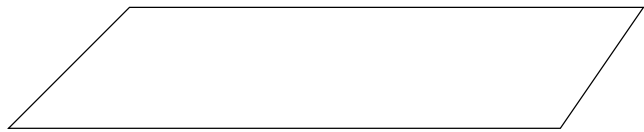
M u u c n c N n
n u n f n y n
n f f u f n n .
func n A u n) n u c n
n n f u . n

nn u f c n n
n u c u

n n c .

n cu





Stieltjes transform

n $\underline{F} \nearrow y$

$$\underline{F} \nearrow -c)I$$

$\frac{1}{z}$

$\frac{1}{z}$

$\frac{1}{z}$

$\frac{1}{z}$

$$z) \left(-\frac{1}{z} + O(z^{-2}) \right), \quad z \rightarrow \infty$$

$$z) \left(-\frac{c}{z} + O(z^{-2}) \right), \quad z \rightarrow \infty$$

$$z) \left(-\frac{c}{a} + \frac{c}{z} + O(z^{-2}) \right), \quad z \rightarrow \infty$$

m_F
 $\frac{1}{z}$
 $f \underline{F}$

$\frac{1}{z}$ $\frac{1}{z}$ $\frac{1}{z}$ $\frac{1}{z}$

Lemma 1 If $z \in \mathcal{U}(F)$, then $m \in \mathcal{M}_F(z)$ satisfies the following.

1. $m \in \mathbb{R}$;

2. $-\frac{1}{m} \in \mathcal{U}(H)$;

3. $z' \in \mathcal{M}(m)$.

Conversely, if m satisfies 1-3, then $z \in \mathcal{M}(z' \in \mathcal{M}(m)) \in \mathcal{U}(F)$.

$$z' \in \mathcal{M}(m) \iff \exists n \in \mathbb{N} \text{ such that } n \cdot z' \in \mathcal{U}(H) \text{ and } n \cdot m \in \mathbb{Z}.$$

Z f yn

a - c) a - c) a - c -))
- c -) a - c) a) a)

n c n n , , c n n

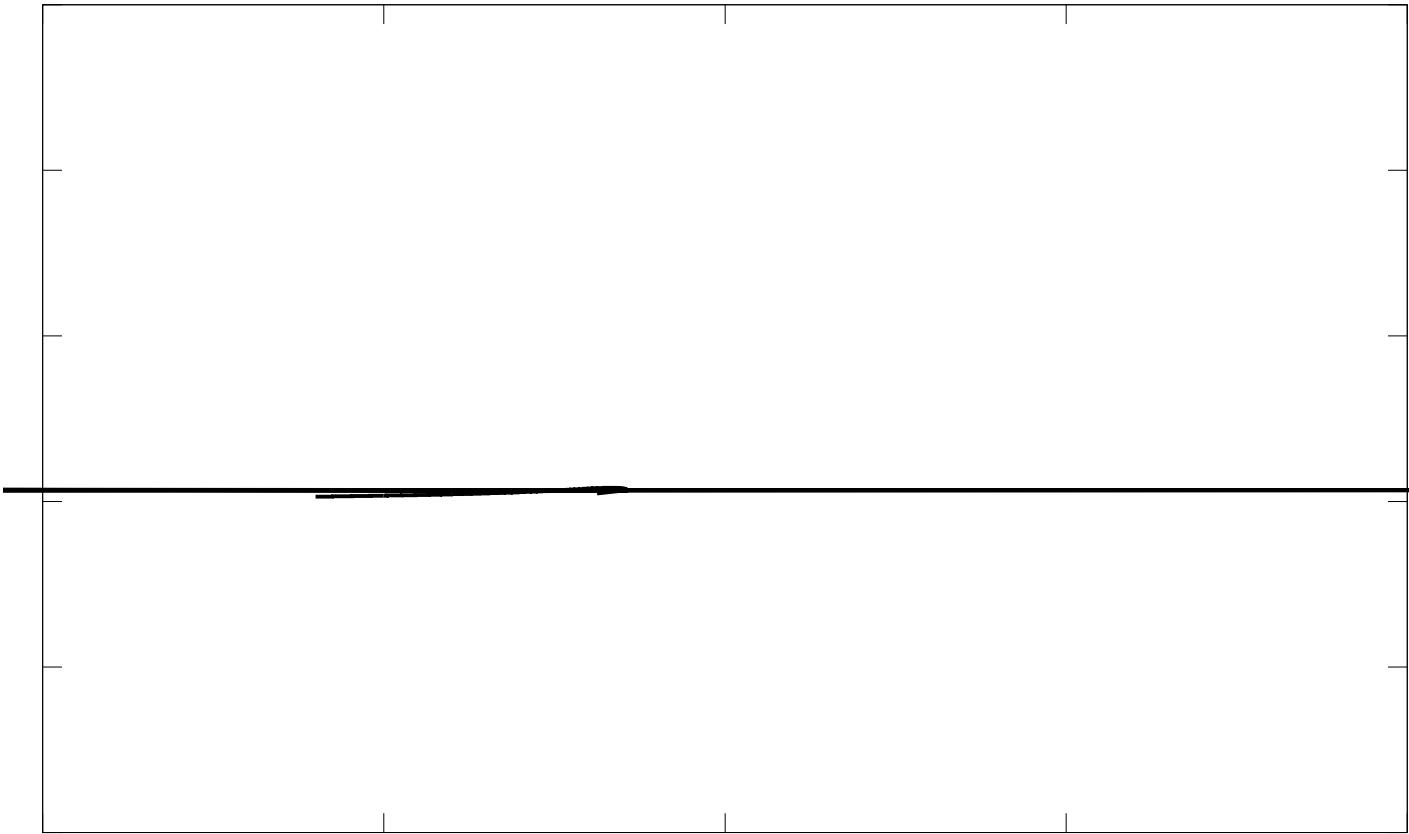
n f u . . n

u F)^c -))))

k n y

k $\frac{c}{k}$ $\frac{c}{k}$ $\frac{a}{a k}$

n f , n u c n f
n u n



Summary

$\frac{N}{N} \rightarrow .$ u c n u n u a n $\frac{N}{M} \rightarrow c,$

n f f c n u n f n n

n n n y n n u n A y
u . c y u n f n

c n f n n c u n
n y n

nc nc ny ny nc n
u , n f nn u f c n
n y c n j n c n nn
u f c n u c n .