

1 - $O(n)$ model

$$W_k^{(g)}(x_1, \dots, x_k) = \frac{1}{\text{number of automorphisms}} \frac{t^V t_3^{n_3} \dots t_d^{n_d} n^L z}{x_1^{j_1+1} \dots x_k^{j_k+1}}$$

ranges over connected genus g discrete surface with k boundaries, built with:

v vertices

n_j j -gons ($j \geq 3$, d , d fixed but arbitrary)

triangles carrying a piece of path forming exactly L loops

a marked j_j -gon ($j_j \geq 1$) with a marked edge as i -th boundary ($1 \leq i \leq k$)

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For $n \in \mathbb{N}$, it admits a representation as a formal hermitian matrix model [1, 6]:

$$\int_{\mathbb{R}^n} d(M, A_1, \dots, A_n) = \int_{\mathbb{R}^n} \overline{dM dA_1 \dots dA_n} \exp \left[-\frac{N}{t} \text{Tr} \left(V(M) + \sum_{i=1}^n \left(\frac{1}{2z} + M \right) A_i^2 \right) \right]$$

$$\text{Tr} \frac{1}{x_i - M} = \sum_{g=0} \frac{N^{2-2g-k}}{t} W_k^{(g)}(K)$$

$$V(x) = \frac{x^2}{2} - \sum_{j=3}^d \frac{t_j}{x^j}$$