TRANSPORT MOMENTS

beyond the leading order

arXiv:1012.3526

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OUTLINE

- Open systems
  - moments

Open systems

- Chaotic cavity with
  $_1 + \; _2 =$
  open channels
- Scattering matrix (E)

ENERGY DEPENDENT

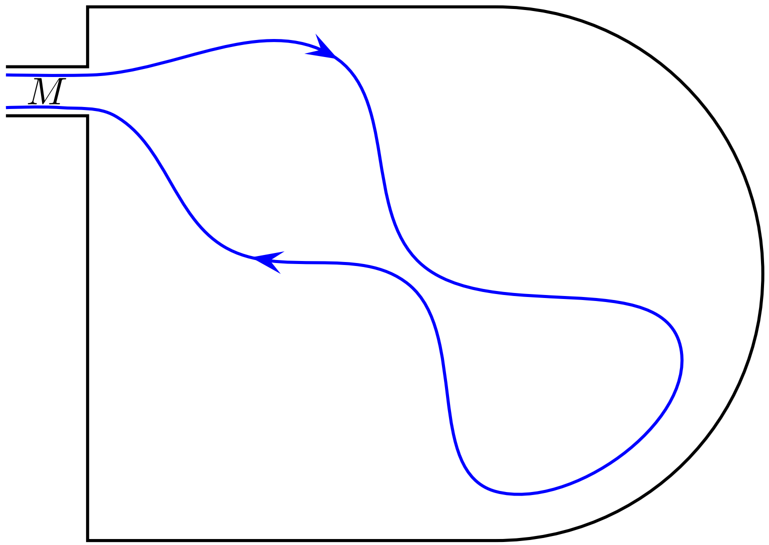
Correlation functions
Semiclassical expression

- Correlation functions

$$C(\quad n) = \frac{1}{\quad}\mathrm{Tr}\left[\ {}^{\dagger}_{-}\quad {}_{+}\right]^{n}$$

- Scattering matrix

$$_{oi}(E)\qquad \sqrt{\frac{}{}} \sum_{\zeta(i\to o)} A_{\zeta} e^{\frac{i}{\hbar} S_{\zeta}},$$



given semiclassically in terms of classical trajectories connecting the corresponding channels

- Action $_{\zeta}$, amplitude A_{ζ}
- is the classical escape rate
- open channels

CORRELATION FUNCTIONS
SEMICLASSICAL EXPRESSION

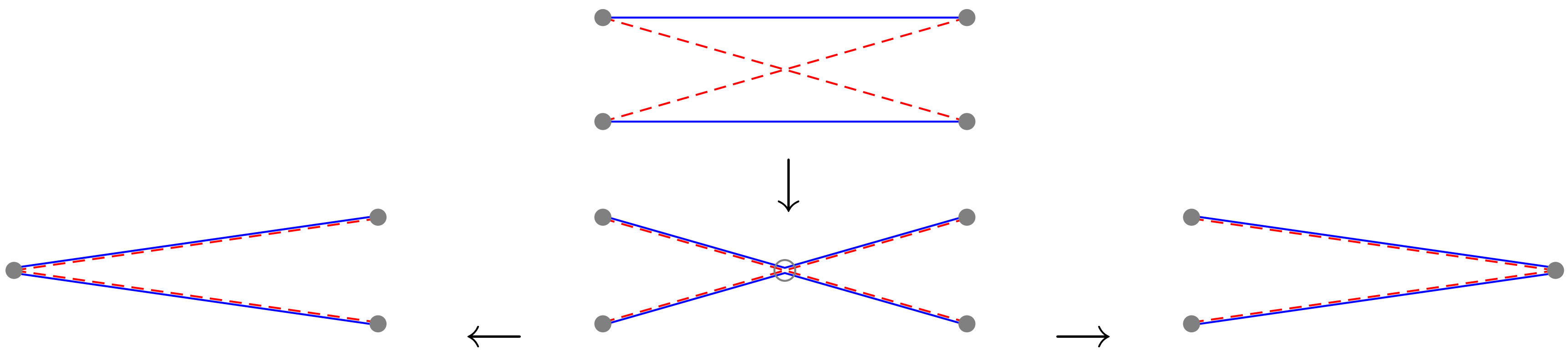
LEADING ORDER
$C(\quad 1)$

- For n = 1, diagonal term

$$C(\quad 1) = \frac{\quad}{2} \sum_{\mathbf{i,o}} \sum_{\zeta(\mathbf{i} \to \mathbf{o}}$$

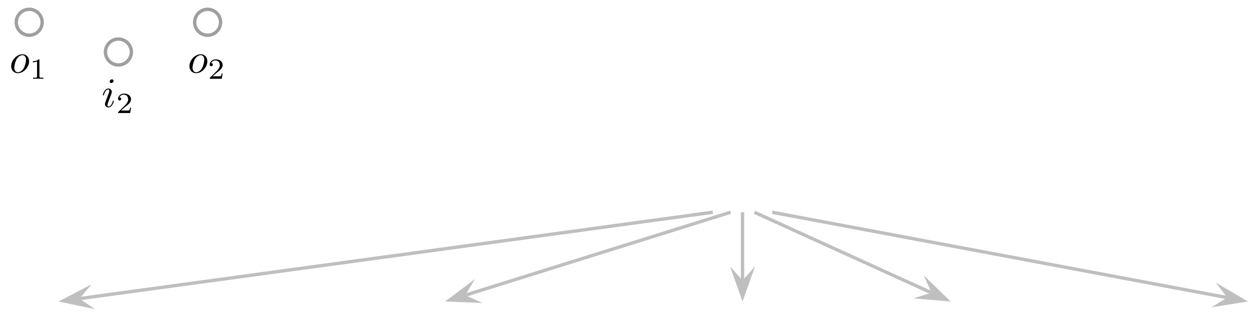
LEADING ORDER
$C(\quad 2)$

- For $n = 2$, can have an encounter



- Can also move encounter into the leads
- For each structure

Semiclassical evaluation
Subtree recursions



Semiclassical evaluation Generating function

- Semiclassical contribution

$$\frac{1}{(1 \quad a)^{n}} \prod_{\alpha=1}^{V} \frac{(1 \quad {}_{\alpha} a)}{(1 \quad a)^{l_{\alpha}}}$$

- Generating function

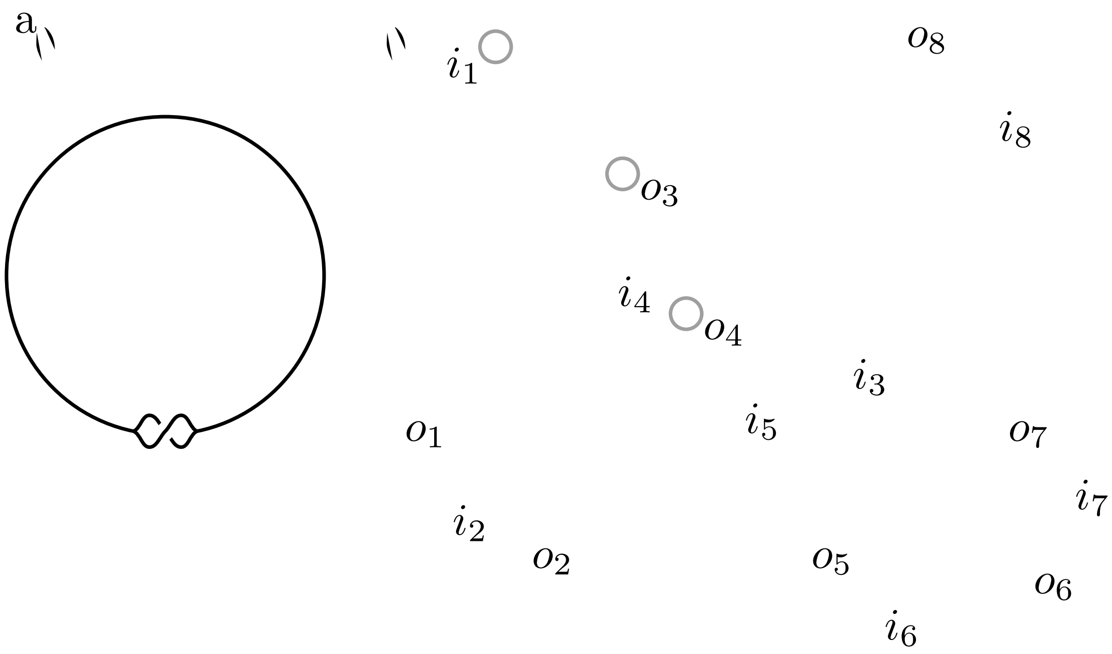
r

S

UR

SIGILLUM

Subleading order graphs



- Odd # of odd nodes, F allows any channel to be the first

$$K_1 = \frac{\tilde{K}_1(p = 1) \quad \tilde{K}_1(p = \quad 1)}{2} \qquad F = r\frac{\mathrm{d}K}{\mathrm{d}r}$$

- Transmission eigenvalues:

$$F(s) = \sum_{\mathbf{n}=1} \tilde{\ }_{\mathbf{n}} s^{\mathbf{n}} \qquad F_1(s) = \frac{s}{(1 \quad s)(1 \quad s + 4 \ s)}$$

with $\quad = \frac{\mathbf{M} \ \mathbf{M}}{\mathbf{M}}$, cf Brouwer and Beenakker, J Math Phys 1996

- Delay times:

$$F(s) = \sum_{\mathbf{n}=1} {}_{\mathbf{n}} \ {}^{\mathbf{n}} s^{\mathbf{n}} \qquad F_1(s) = \frac{1 \quad 3s \quad 1 \quad 6s + s^2}{2(1 \quad 6s + s^2)}$$

SIGILLUM

NEXT ORDER

ORTHOGONAL CASE

ANDREEV BILLIARDS

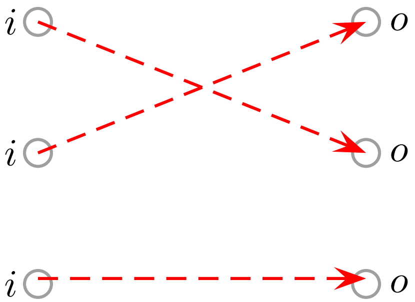
- Replace lead by superconducting block
  - electrons reflected back as holes
- Density of states Ihra et al, EPJB 2001
  - renormalised by average density

A

Nonlinear statistics

Two traces

a)



Conclusions

- Leading order, we obtained semiclassically
  - moments of the delay times JPA (2010) **43** 035101
  - density of states of Andreev billiards PRL (2010) **104** 027001; arXiv:1004.1327

- Next two corrections, we can obtain arXiv:1012.3526
  - transmission eigenvalues
  - delay times
  - Andreev billiards
  - nonlinear statistics