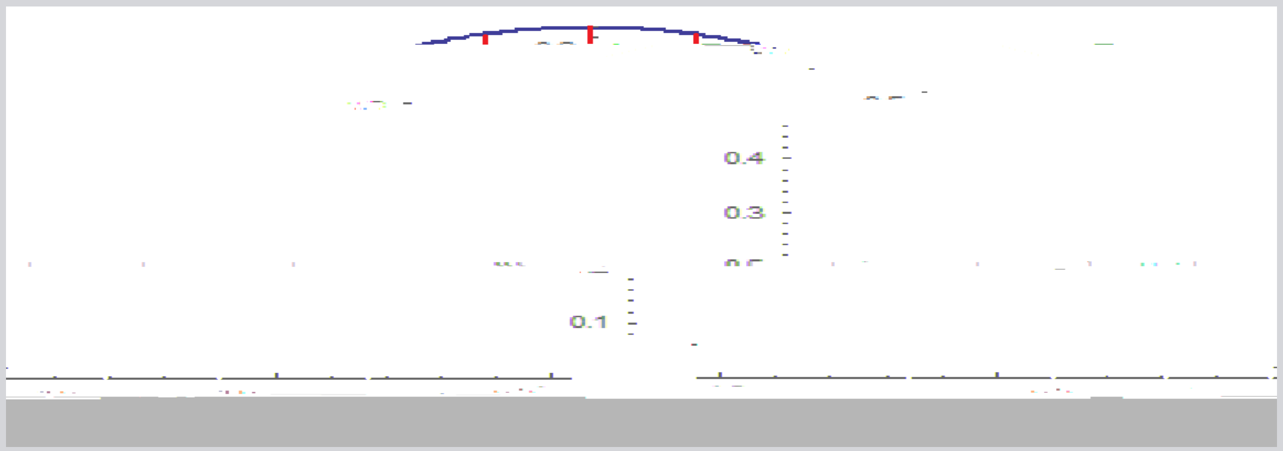
Global rigid[redacted]

Cla ical GUE eigenvalue location

Let $_1 \leq \ _2 \leq \cdots \leq$ be the ord[redacted]
ize $\times$, normalized [redacted]
converge to a emi-circle law on $[-1, 1]$. We [redacted] cla ical location $_1, \ _2, \ldots, \ \in [-1, 1]$ of the eigenvalue by

$$\frac{2}{\ } \int_{-1} \sqrt{1 - \ ^2} = \frac{\ }{\ }, \qquad = 1, \ldots, \ .$$



Global rigidity

What can we ay for large about the di tribution of the normalized maximal fluctuation of eigenvalue (cf. Bourgade-Erdos-Yau)

$$:= \max_{=1,\ldots,} \frac{2}{} \sqrt{1-{}^2}\,| - |\ ?$$

Global rigidity in the GUE

Upper bound for generalized Wigner matrices (Erdos-Yau-Yin '12)

$$\geq \frac{(\mathrm{l\ g}\quad)^{\mathrm{l\ g\ l\ g}}}{\qquad} \leq \quad \mathrm{e\ p} - (\mathrm{l\ g}\quad)^{\prime\,\mathrm{l\ g\ l\ g}}$$

Lower bound for GUE (Gustavsson '05)

$$2\quad\overline{2}\quad\sqrt{1-\quad^{2}}\frac{}{\sqrt{\mathrm{l\ g}}}(\quad - \quad) \to \quad (0,1)$$

for $\leq \quad \leq (1- \quad)$, which implie (non-optimal) lower bound for

.

Global rigidity in unitary invariant en emble

Extreme of log-correlated field

The problem i then to tudy extrema of the log-correlated field .

Extrema of uch proce e have been tuNG g

Multiplicative chao

Powerful tool to tud[redacted]eory of multiplicative chao

- General theory (Kahane '85, Rhodes-Vargas '1 , Berestycki '15)
- Applie[redacted]yodorov-

Upper bound estimate

The upper bound for can be obtained using an elementary one-moment method

Global rigidity in unitary invariant en emble

Upper bound e timate

- i a Hankel determinant with di continuou weight , and [redacted] a ymptotic for uch Hankel [redacted] known for (Its-Krasovsky '08 for GUE, Charlier '18 for one-cut regular unitarxYHv

Global rigidity in

Global rigidity in unitary invariant ensembles

Lower bound estimates

Optimal lower bound estimates are much harder to obtain, and require to investigate the log-correlated structure of

Log-correlated structure

It is well-known (Johansson '98) that behave for l

Multiplicative chao

Maximum of the eigenvalue counting function

For tudying the maximum of , we prove that the random mea ure

$$ = \quad , \quad \in $$

converge weakly in di tribution to a multiplicative chao mea ure which can be formally written a (cf. Kahane '85, Rhodes-Vargas '10, Berestycki '17, Berestycki-Webb-Wong '17)

Multiplicative chao

freezing tran ition

Another con equence of the multiplicative chao convergence i that

in probability.

In the phy ic literature, thi i called a freezing tran ition of the random energy land cape.

Exponential moment e timate

Two merging ingularitie

a , where the error term i uniform for

, for ufficiently mall.

Method of proof

We prove thi u ing a imilar method than the one u ed for Toeplitz determinant with merging Fi her-Hartwig ingularitie (C-Krasovsky '15) and Hankel determinant with merging root ingularitie (C-Fahs '16), ba ed dti nami ti ogmeth-nv-nBtCjhQ7A8xQZzzG ! vQBtCjhQ7A8xQ

-dependent

A ume that $\quad = \quad$ i a equence of function which are analytic and [redacted]
too fa t with .

$$\mathrm{l\ g} \quad (\ _1, \ _2; \ _1, \ _2; \quad) = \mathrm{l\ g} \quad (\ _1, \ _2; \ _1, \ _2; 0)$$

$$+ \qquad\qquad + \frac{1}{2} (\quad)^2 + \sum_{=1}^{2} \frac{}{\sqrt{2}} \sqrt{1 - \ ^2} \quad (\ ) + (1),$$

a $\quad \to \infty$, uniformly for $(\ _1, \ _2)$ in any fixed [redacted]
$(-1, 1)^2$, where

$$(\quad = \qquad (\ 5 ?$$

Exponential moment e timate

Finally, we need [redacted]

ingularity tend[redacted]

bound [redacted].

Singularity clo e to the edge [redacted]

$$\mathrm{l\ g}\ \frac{(\ ;\ \ ;0)}{(\ ;0;0)} = \qquad (\ ) + \ \mathrm{l\ g}\ + \ \mathrm{l\ g}(1 - \ \ ) + \ (1),$$

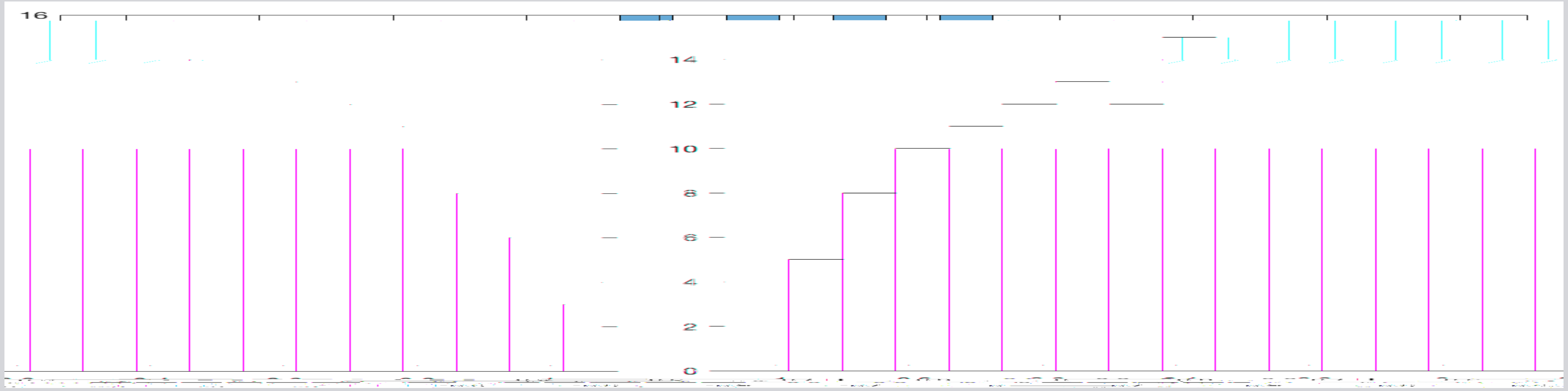
a , with the error term uniform for all , with

ufficiently large.

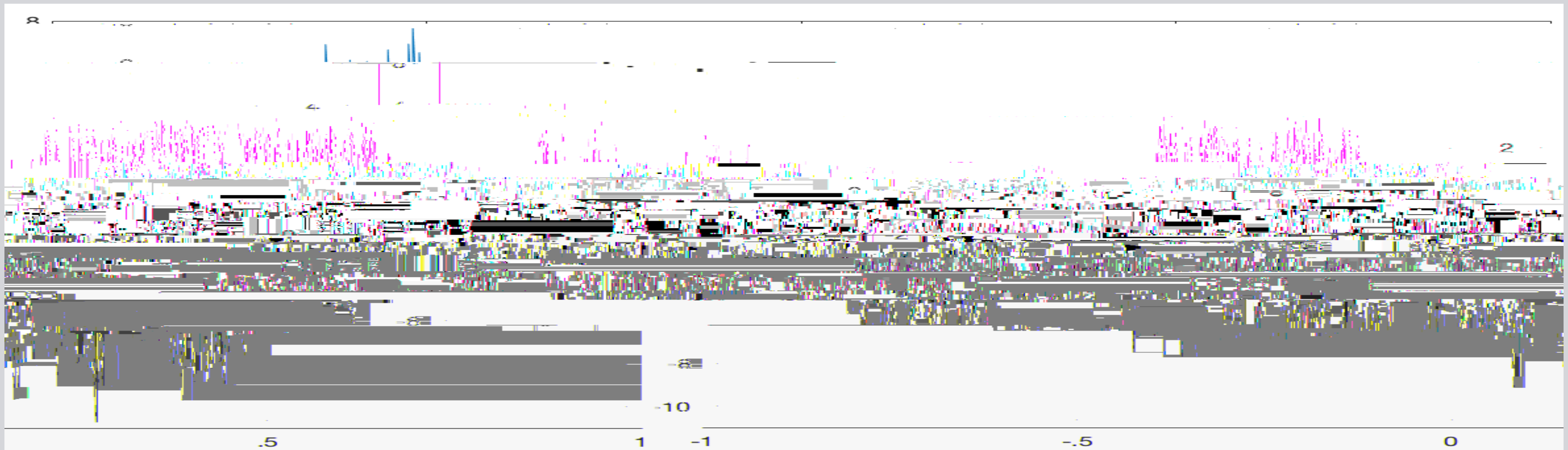
Summary of the method

1. Hankel determinant a ymptotic
   - Convergence of to a multiplicative chao mea ure
   - E timate for -thick point
   - E timate fọr the lower bound of
2. Hankel determinant a ymptotic
   - E timate for the upper bound of

Hi togram of GUE eigenvalue for $= 300$

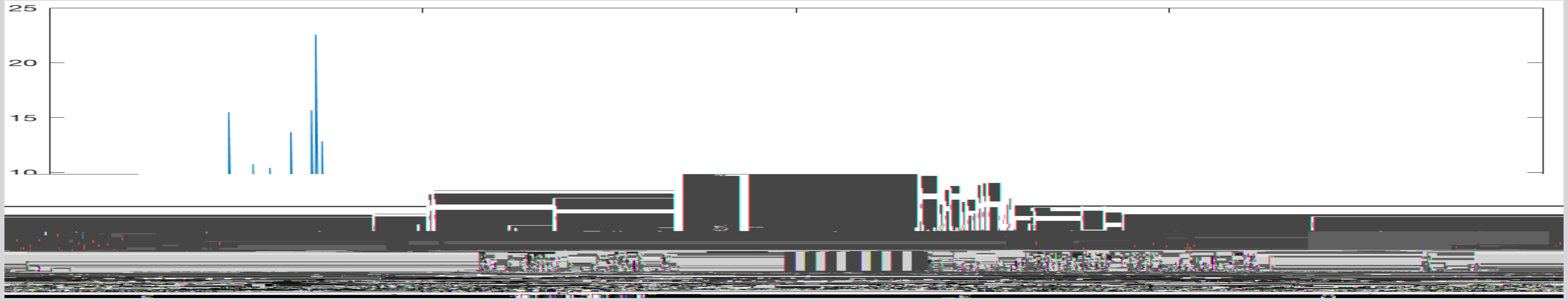


Normalized eigenvalue counting function for $= 300.$

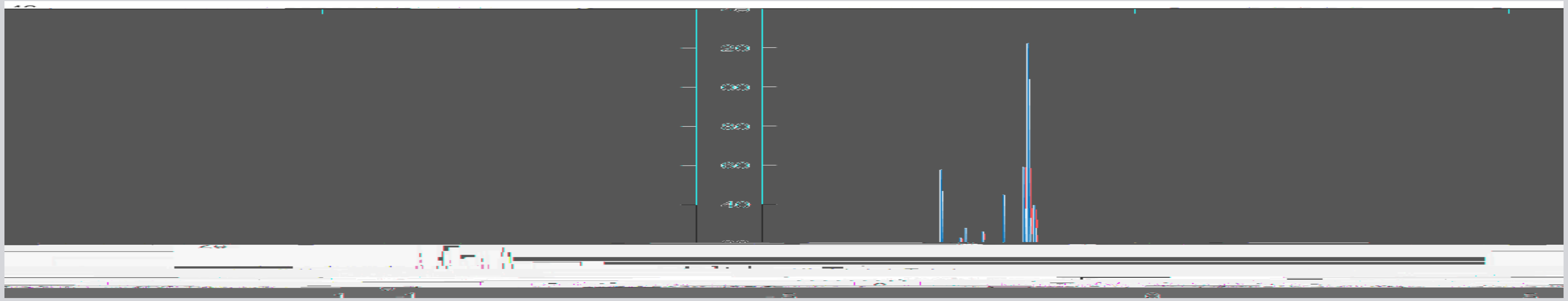


Simulation

with



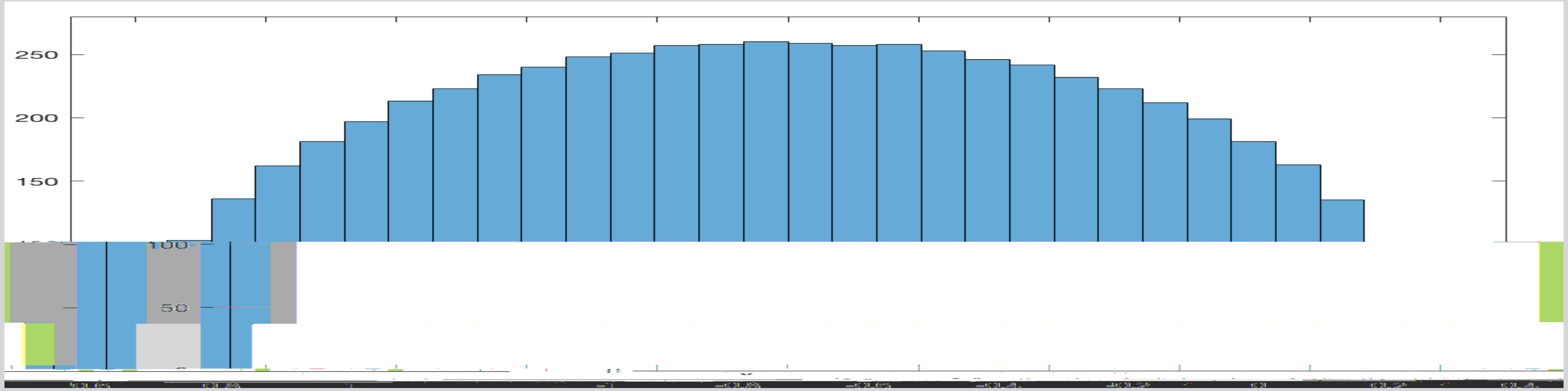
with



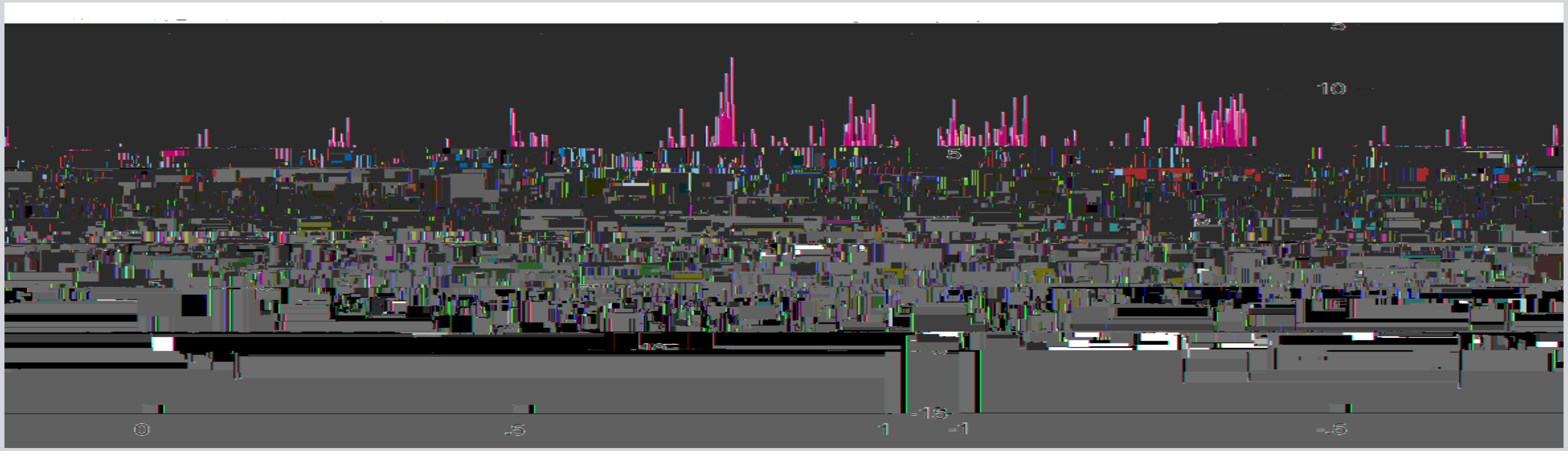
with

Simulation

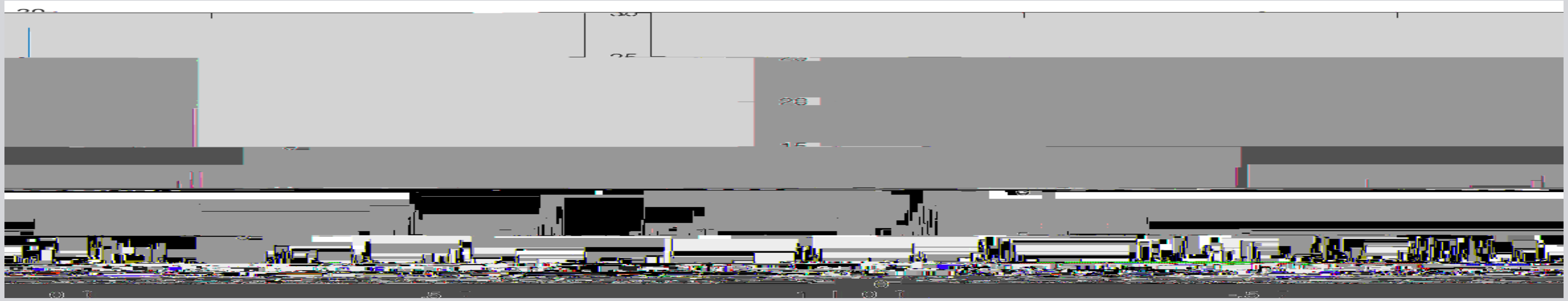
Hi togram of GUE eigenvalue for $= 300$



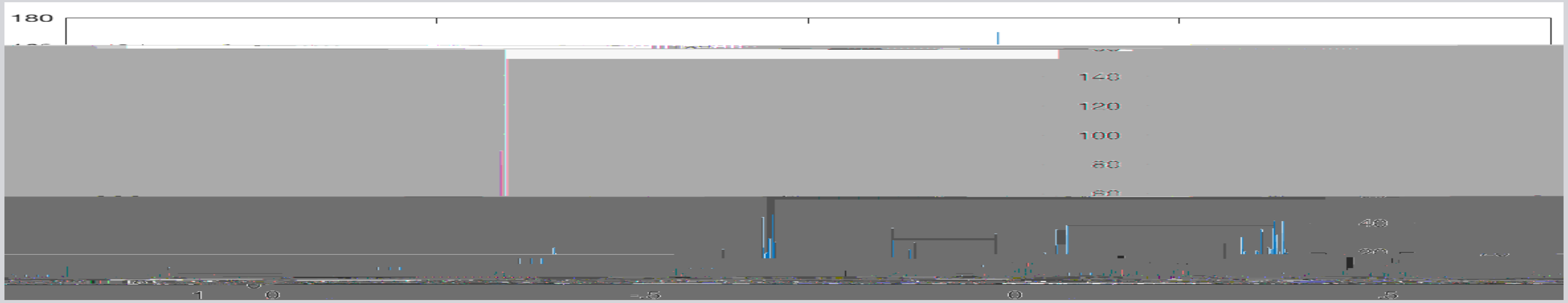
Normalized eigenvalue counting function for $= 300.$



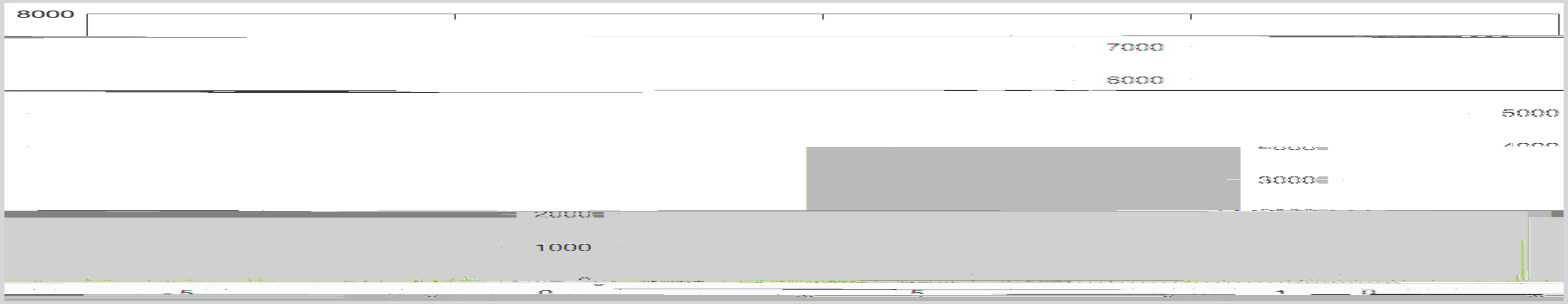
$\frac{(\ )}{(\ )}$ with $= 0.3$



$\frac{(\ )}{(\ )}$ with $= 0.5$



with



Thank you for your attention!